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The homotopy groups $\pi_*(L_2 V(0) \wedge T(k))$

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Abstract. Let $V(0)$ and $T(k)$ denote the mod p Moore spectrum and the Ravenel spectrum at a prime p, respectively. We determine the homotopy groups $\pi_*(L_2 V(0) \wedge T(k))$ for $k \geq 2$ and $p > 2$. This is done by determining the chromatic E_1-term $H(k)^* M_1^1$, which is obtained by using only two key lemmas: one is to define the Miller-Ravenel-Wilson elements and the other is to give a one dimensional element ζ.

1. Introduction and the statement of results

Let $T(k)$ denote the Ravenel ring spectrum at a prime p, which is characterized by the Brown-Peterson homology $BP_*(T(k)) = BP_*[t_1, t_2, \dots, t_k] \subset BP_*(BP) = BP_*[t_1, t_2, \dots]$, where $BP_* = \boldsymbol{Z}_{(p)}[v_1, v_2, \dots]$. Note that $T(0) = S^0$. Then the homotopy groups $\pi_*(T(k))$ are, in a sense, an approximation of the homotopy groups $\pi_*(S^0)$ of spheres. For the Bousfield localization functor L_n on the stable homotopy category with respect to $v_n^{-1}BP$, the homotopy groups $\pi_*(L_n S^0)$ are also an approximation of $\pi_*(S^0)$. Both of the homotopy groups are considerably easier to compute than the homotopy groups of spheres. In this paper we determine the homotopy groups of $L_2 V(0) \wedge T(k)$ for each $k \geq 2$ at an odd prime p, where $V(0)$ denotes the mod p Moore spectrum. These groups are computed by the Adams-Novikov spectral sequence and the chromatic spectral sequence. The E_1-terms of the chromatic spectral sequence are $H(k)^* M_1^0$ and $H(k)^* M_1^1$, where $H(k)^* M = \mathrm{Ext}^*_{BP_*(BP)}(BP_*, M \otimes_{BP_*} BP_*(T(k)))$. If the prime p is odd, then the Adams-Novikov spectral sequence collapses from the E_2-term, and so it suffices to determine the chromatic E_1-terms to obtain the module structure of $\pi_*(L_2 V(0) \wedge T(k))$. Ravenel determined $H(k)^* M_1^0$ (*cf.* [4]) and we determine $H(k)^* M_1^1$ here not only for an odd prime p but also for the prime 2. We note that our computation for $H(k)^0 M_1^1$ also works in the case where $k = 1$ and $p > 2$. For the case $k = 0$, it is determined in [10] and [6] if $p > 3$, in [9] if $p = 3$ and in [8] if $p = 2$, which show that the computation is very

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