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Singularities of non-degenerate 2-ruled hypersurfaces in 4-space

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ABSTRACT. We study singularities of 2-ruled hypersurfaces in Euclidian 4-space. After defining a non-degenerate 2-ruled hypersurface we will give a necessary and sufficient condition for such a map germ to be right-left equivalent to the cross cap $\times$ interval. The behavior of a generic 2-ruled hypersurface map is also discussed.

1. Introduction

The study of ruled surfaces in $\mathbf{R}^3$ is a classical subject in differential geometry and ruled hypersurfaces in higher dimensions have also been studied by many authors. Although ruled hypersurfaces have singularities in general, there have been very few studies of ruled hypersurfaces with singularities. Recently Izumiya and Takeuchi [3] showed that every singularity that appears for some generic C^∞-map of a surface into 3-space occurs for some generic ruled surface in $\mathbf{R}^3$, and vice versa.

A 2-ruled hypersurface in $\mathbf{R}^4$ is a one-parameter family of planes in $\mathbf{R}^4$. This is a generalization of ruled surfaces in $\mathbf{R}^3$. In this paper, we first define non-degenerate 2-ruled hypersurfaces in $\mathbf{R}^4$ and give a necessary and sufficient condition for a non-degenerate 2-ruled hypersurface germ in $\mathbf{R}^4$ to be right-left equivalent to the cross cap $\times$ interval (Theorem 2.5). Furthermore, we show that the singularities of generic 2-ruled hypersurfaces are cross cap $\times$ interval (Theorem 5.3). Since any singularity of a generic smooth map of a 3-manifold into $\mathbf{R}^4$ is the cross cap $\times$ interval, the singularities of generic 2-ruled hypersurfaces are the same as those of generic C^∞-maps of 3-manifolds into $\mathbf{R}^4$.

The paper is organized as follows. In §2 we define non-degenerate 2-ruled hypersurfaces as an analogue of classical noncylindrical ruled surfaces. Classical noncylindrical ruled surfaces are those whose rulings always change directions and non-degenerate 2-ruled hypersurfaces will be defined in the same way. Then we present the main theorem (Theorem 2.5). In §3 we briefly review the properties of the classical striction curve and generalize them to non-degenerate 2-ruled hypersurfaces. It is quite remarkable that the striction curve

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