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On upper and lower bounds of rates of decay for nonstationary Navier-Stokes flows in the whole space

Dedicated to Professors Masayasu Mimura and Takaaki Nishida on the occasion of their 60th birthdays

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Abstract. Upper and lower bounds of rates of decay in time are studied for nonstationary Navier-Stokes flows in $\mathbf{R}^n$ with the aid of Besov spaces in which the solutions exist for all time. It is shown that there is a Besov space, with norm $\|\cdot\|$, in which the solution $u(t)$ satisfies the estimate $0 < c \le \|u(t)\| \le c'$ for all $t \ge 0$ provided the initial velocity satisfies suitable moment conditions. Our argument is then applied to the analysis of flows with cyclic symmetry, introduced by Brandolese [3], and it is shown that these flows decay more rapidly in space and time than proved in [3]. However, the existence of a lower bound as mentioned above remains open for such flows.

1. Introduction and results

This paper continues the previous works [4, 8, 9, 11] on the asymptotic behavior as $t \to \infty$ of nonstationary Navier-Stokes flows $u = (u_j)_{j=1}^n$ in $\mathbf{R}^n$, $n \ge 2$, which are governed by the integral equation:

$$\text{(IE)} \qquad u(t) = e^{-tA}a - \int_0^t e^{-(t-s)A} P\nabla \cdot (u \otimes u)(s)ds, \qquad t \ge 0.$$

Here, $\nabla = (\partial_1, \dots, \partial_n)$, $\partial_j = \partial/\partial x_j$, $\nabla \cdot (u \otimes u) = (\sum_j \partial_j(u_j u_k))_{k=1}^n$; $u = (u_j)_{j=1}^n$ is unknown velocity and $a = (a_j)_{j=1}^n$ is a given initial velocity, both of which are required to satisfy the divergence-free condition

$$\nabla \cdot u = 0, \qquad \nabla \cdot a = 0.$$

$A = -\Delta$ is the Laplacian on $\mathbf{R}^n$, $e^{-tA}a = E_t * a$ is the convolution with the heat kernel

$$E_t(x) = (4\pi t)^{-n/2} e^{-|x|^2/4t},$$

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