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A strong limit theorem expressed by inequalities for the sequences of absolutely continuous random variables

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Abstract. Let $\{X_n, n \geq 1\}$ be an arbitrary sequence of dependent absolutely continuous random viariables, $\{B_n, n \geq 1\}$ be Borel sets on the real line, and $I_{B_n}(x)$ be the indicator function of B_n. In this paper, the limit properties of $\{I_{B_n}(X_n), n \geq 1\}$ are studied, and a kind of strong limit theorem represented by inequalities with random bounds is obtained.

1. Introduction

Let $\{X_n, n \geq 1\}$ be a sequences of absolutely continuous random variables on the probability space $(\Omega, \mathscr{F}, P)$ with the joint density function $g_n(x_1, \dots, x_n)$, $n = 1, 2, \dots$. Let $f_k(x_k)$, $k = 1, 2, \dots$, be an arbitrary sequence of density functions, and call $\prod_{k=1}^{n} f_k(x_k)$ the reference product density. Let

$$r_n(\omega) = \begin{cases} \left[\prod_{k=1}^{n} f_k(X_k)\right] \Big/ g_n(X_1, \dots, X_n) & \text{if the denominator} > 0; \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where ω is a sample point. In statistical terms, $r_n(\omega)$ is called the likelihood ratio, which is of fundamental importance in the theory of testing the statistical hypotheses (cf. [**1**, p. 483]; [**3**, p. 388]). Let

$$r(\omega) = -\liminf_{n} \frac{1}{n} \ln r_n(\omega) \tag{2}$$

with $\ln 0 = -\infty$. $r(\omega)$ is called asymptotic log-likelihood ratio. Obviously, $r_n(\omega) \equiv 0$ if

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