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Selberg zeta functions for cofinite lattices acting on line bundles over complex hyperbolic spaces

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Abstract. For a line bundle over a finite volume quotient of the complex hyperbolic space, we write down an explicit trace formula for an admissible function lying in the Harish-Chandra p-Schwartz space $\mathscr{C}^p(G)$, $0 < p < 1$, we apply it to a suitable admissible function in order to discuss the analytic continuation of the associated Selberg zeta function.

1. Introduction

Let Y be a finite volume non compact locally symmetric space of negative curvature, that is $Y = \Gamma\backslash G/K$ where G is a real semi-simple Lie group of $\mathbf{R}$-rank one, K is a maximal compact subgroup of G, $\Gamma \subset G$ a cofinite discrete subgroup of G.

In 1956, for $G = SL(2, \mathbf{R})$, $G/K = H$ the upper half plane and Γ a discrete subgroup of G, Atle Selberg in his famous paper [10] introduced a function $Z_\Gamma(s)$ of one complex variable, so called Selberg zeta function and showed that the location and the order of the zeros of this function gives information on the topology of the manifold $Y = \Gamma\backslash H$ as well as on the spectrum of the associated Laplace-Beltrami operator.

In 1977, R. Gangolli [7] extended the result of Selberg to a general G of rank one and $Y = \Gamma\backslash G/K$ compact by constructing Selberg type Zeta function for this general case. Two years after, the same author jointly with G. Warner [6] treated analogously the case where $\Gamma\backslash G$ is not compact but of finite volume for a general G of rank one. However, for technical reasons, they avoided the case where $G = SU(2n, 1)$. Their work was based on the explicit Selberg trace formula written down by G. Warner for $G = SU(2n+1, 1)$ in his survey paper [15]. This zeta function provides some topological data on the manifold $\Gamma\backslash G/K$ as well as some spectral information. That is, the class one spectrum induced from the trivial representation of K contained in $L^2_{disc}(\Gamma\backslash G)$.

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