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Extendibility and stable extendibility of vector bundles over lens spaces mod 3

Dedicated to the Memory of Professor Masahiro Sugawara

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ABSTRACT. In this paper, we prove that the tangent bundle $\tau(L^n(3))$ of the $(2n+1)$-dimensional mod 3 standard lens space $L^n(3)$ is stably extendible to $L^m(3)$ for every $m \geq n$ if and only if $0 \leq n \leq 3$. Combining this fact with the results obtained in [6], we see that $\tau(L^2(3))$ is stably extendible to $L^3(3)$, but is not extendible to $L^3(3)$. Furthermore, we prove that the t-fold power of $\tau(L^n(3))$ and its complexification are extendible to $L^m(3)$ for every $m \geq n$ if $t \geq 2$, and have a necessary and sufficient condition that the square ν^2 of the normal bundle ν associated to an immersion of $L^n(3)$ in the Euclidean $(4n+3)$-space is extendible to $L^m(3)$ for every $m \geq n$.

1. Definitions and results

The extension problem is one of the fundamental problems in topology. We study the problem for F-vector bundles over standard lens spaces mod 3, where F is either the real number field R or the complex number field C.

First, we recall the definitions of extendibility and stable extendibility according to [12] and [2]. Let X be a space and A be its subspace. A k-dimensional F-vector bundle ζ over A is said to be extendible (respectively stably extendible) to X, if there is a k-dimensional F-vector bundle over X whose restriction to A is equivalent (respectively stably equivalent) to ζ as F-vector bundles, that is, if ζ is equivalent (respectively stably equivalent) to the induced bundle $i^*\alpha$ of a k-dimensional F-vector bundle α over X under the inclusion map $i : A \to X$. For simplicity, we use the same letter for an F-vector bundle and its equivalence class, and use a non-negative integer k for the k-dimensional trivial F-vector bundle.

For a non-negative integer n and an integer $q > 1$, let $L^n(q)$ denote the $(2n+1)$-dimensional standard lens space mod q and $L_0^n(q)$ its $2n$-skeleton (cf. [3], [4] and [11]). For a positive integer n, let η_n stand for the canonical C-line

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