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Cobordism group of Morse functions on manifolds

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Abstract. The n-dimensional cobordism group of Morse functions on manifolds is defined by using maps into $\mathbf{R} \times [0, 1]$ with only fold singularities. In this paper, we show that in the un-oriented case it is a direct sum of the n-dimensional cobordism group and a certain number of infinite cyclic groups. In the oriented case a finite cyclic group $\mathbf{Z}_2$ is further added when $n = 4k + 1$.

1. Introduction

The n-dimensional oriented cobordism group $\mathscr{M}_n$ of Morse functions was introduced in Ikegami–Saeki [2], where we used a different notation. The purpose of this paper is to determine the structures of $\mathscr{M}_n$ and the n-dimensional un-oriented cobordism group $\mathscr{N}_n$ of Morse functions. We use "elimination of cusps" [4] and "semi-characteristics" [5]. Note that we showed in [2] that $\mathscr{M}_2$ is an infinite cyclic group, by a different method.

For the cobordism theory of smooth maps, Thom [8] showed that the cobordism group of embeddings is isomorphic to a homotopy group of a certain Thom complex by using the Pontrjagin-Thom construction. Wells [10] studied the cobordism group of immersions in a similar way. Rimányi and Szűcs [6] extended these results to the cobordism group of maps with singularities by using the notion of a τ-map.

Usually a cobordism group is computed by using the method of algebraic topology as a certain homotopy group of a Thom complex. However, in this paper the cobordism group of Morse functions is completely determined in a geometric way.

Recently Saeki [7] considered another kind of n-dimensional cobordism groups of special Morse functions and got a relation with the h-cobordism group of homotopy n-spheres for $n \geq 6$.

The paper is organized as follows. In § 2 we recall the precise definition of the n-dimensional cobordism group of Morse functions and state our main theorem. In § 3 defining fold and cusp points of a smooth map into $\mathbf{R}^2$ to-

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