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Some acyclic relations in the lambda algebra

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Dedicated to the Memory of Professor Masahiro Sugawara

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Abstract. We consider the relations $\omega\gamma = 0 \in \Lambda$, and show that if $\omega\alpha = 0$ then $\alpha = \gamma\beta$ for some β. These relations give the acyclic chain complex $\Lambda \xrightarrow{\gamma} \Lambda \xrightarrow{\omega} \Lambda$. We consider various cases, e.g. $\omega = \lambda_n$ and $\gamma = \lambda_{2n+1}$. Especially, we consider the case $\omega = w_n = d\lambda_n$ for $n = 2^{e+r} + 2^e - 1$, where $\gamma = (h_{e+r})^r$.

1. Introduction

Consider the stable homotopy groups of the sphere $\pi_*(S^0)$ localized at prime 2. We have the 2-local Adams spectral sequence converging to $\pi_*(S^0)$ with E_2-term $\mathrm{Ext}_A^{s,t}(\mathbf{Z}/2, \mathbf{Z}/2) = H^{s,t}(\Lambda)$ by [2]. Moreover, Λ contains a subcomplex $\Lambda(n)$ whose cohomology is the E_2-term of the unstable Adams spectral sequence converging to the 2-component of the unstable homotopy groups of S^n. There are corresponding p-local versions of Λ algebra that we will not consider.

The lambda algebra Λ (at the prime $p = 2$) is a bigraded $\mathbf{Z}/2$-algebra with generators $\lambda_n \in \Lambda^{1,n+1}$ $(n \geq 0)$ and relations

$$\lambda_i \lambda_{2i+1+n} = \sum_{j \geq 0} \binom{n-1-j}{j} \lambda_{i+n-j} \lambda_{2i+1+j} \qquad (i, n \geq 0) \tag{1}$$

with differential

$$d\lambda_n = \sum_{j \geq 1} \binom{n-j}{j} \lambda_{n-j} \lambda_{j-1} \qquad (n \geq 0). \tag{2}$$

We refer to [9] for these relations and [2, 5] for that d is a well-defined endomorphism of Λ. For a sequence $I = (n_1, n_2, \ldots, n_s)$ of non-negative integers, a monomial $\lambda_I = \lambda_{n_1} \lambda_{n_2} \ldots \lambda_{n_s}$ is said to be admissible if $2n_i \geq n_{i+1}$ for $1 \leq i \leq s-1$. The admissible monomials form an additive basis of Λ by [2, 5]. $\Lambda(n) \subset \Lambda$ is the subcomplex spanned by the admissible monomials with

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