HIROSHIMA MATH. J.
34 (2004), 1–19

Kernels of derivations in positive characteristic

Shun-Ichiro OKUDA

(Received January 17, 2003)
(Revised October 23, 2003)

ABSTRACT. Let R be an integral domain that is finitely generated over a field k. Let $D: R \to R$ be a derivation over k. Our aim is to compute $\operatorname{Ker} D$.

Under the assumptions that the characteristic of k is zero, D is locally nilpotent and $\operatorname{Ker} D$ is finitely generated over k, Essen gave an explicit algorithm based on the exponential of the derivation. In this paper we give an analogous algorithm in the positive characteristic case using a truncated version of the exponential. It does not require the nilpotence of D. We give several computational examples of application of our algorithm.

Also using higher derivations, we obtain a word-by-word translation of Essen's formula to positive characteristics.

1. Introduction

Let R be an integral domain that is finitely generated over a field k. Let $D: R \to R$ be a derivation over k. The subject of this paper is to compute the kernel $\operatorname{Ker} D$ in positive characteristic.

When the characteristic of k is zero, several techniques to compute the derivation kernel are known. Essen gave one of such techniques in the case that D is locally nilpotent and that $\operatorname{Ker} D$ is finitely generated over k ([6], [7, Corollary 1.3.23]). The technique is based on the exponential of the derivation. Note that in characteristic 0, some derivation kernels are not finitely generated (counter-examples to Hilbert's Fourteenth Problem), even if the derivations are locally nilpotent (see [4], [8], [9], [14], [15]). Essen's algorithm does not work for these derivations.

In this paper we give an algorithm to compute the derivation kernel in the positive characteristic case, inspired by Essen's algorithm. And using our algorithm, we can compute the kernel for any derivation, without assuming the nilpotency of the derivations. The key points of our algorithm are that we regard the derivation kernel as a $k[R^p]$-module, where $k[R^p]$ is a sub k-domain of R generated by $\{x^p \mid x \in R\}$ and that we compute the generators of the kernel as the $k[R^p]$-module using the truncated version of the exponential of the der-

2000 *Mathematics Subject Classification.* 13N15.
Key words and phrases. the kernel of derivations in positive characteristic.