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On a nonlinear diffusion system with resource-consumer interaction

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ABSTRACT. This article is devoted to the study of a resource-consumer type reaction-diffusion system arising in chemistry, biology and in other applied sciences. We prove that the problem is well-posed and describe the large time behavior of the solutions. A key ingredient is to obtain a uniform in time L^∞-bound for the solutions. We also present numerical simulations describing the transient behavior of the solution which show very unstable interfaces.

1. Introduction

Among a lot of reaction-diffusion (RD) equations, a class of RD equations with consumer and resource interaction have been thoroughly investigated by many authors. A typical but suggestive example is the following two component system where u and v act as a consumer and its resource, respectively:

$$\begin{cases} u_t = d_u \Delta u + u^m v, \\ v_t = d_v \Delta v - u^m v, \end{cases} \tag{1.1}$$

where d_u and d_v are the diffusion coefficients of u and v and m is a positive integer. The main results concern the well-posedness of the parabolic problems, L^∞-bounds on solutions which do not depend on time and their asymptotic behavior ([Ali, Mas, HaYo, Hos, Kan] for instance). A characteristics of this system under the zero-flux boundary condition is that the spatial average of $u+v$ is conserved in time. It is shown that

$$\lim_{t\to\infty}(u(t), v(t)) = (\langle u(0) + v(0)\rangle, 0),$$

where $\langle f\rangle$ is the spatial average of f.

In view of the result on the asymptotic behavior of the solutions above, one used to believe that resource-consumer systems without feeding process such as (1.1) are not interesting from the pattern formation viewpoint. However, recent numerical simulations have revealed that it is not necessarily

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