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The structure of the general chromatic E_1-term $\mathrm{Ext}^0_{\Gamma(2)}(BP_*, M_2^1)$ at the prime 2

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Abstract. Ravenel [8] has introduced p-local spectra $T(m)$ for $m \geq 0$. The Adams-Novikov E_2-term converging to $\pi_*(T(m))$ is isomorphic to

$$\mathrm{Ext}^*_{\Gamma(m+1)}(BP_*, BP_*),$$

where $\Gamma(m+1) = BP_*[t_{m+1}, t_{m+2}, \dots]$, and thus we may follow the chromatic method introduced in [4] to compute the E_2-term. One of the crucial point is to determine the Ext groups $\mathrm{Ext}^*_{\Gamma(m+1)}(BP_*, M_s^n)$. In particular $\mathrm{Ext}^0_{\Gamma(m+1)}(BP_*, M_2^1)$ has already been known except for $p = 2$ and $m = 1$. In this paper we will give the explicit description of the last unknown case.

1. Introduction

The homotopy groups of Ravenel spectrum $T(m)$ give information on the homotopy groups of spheres using "the method of infinite descent", which was the main subject of [8] Chapter 7. Its BP-homology group is given by $BP_*(T(m)) \cong BP_*[t_1, \dots, t_m]$. The Adams-Novikov E_2-term for $T(m)$ is

$$\mathrm{Ext}^*_{BP_*(BP)}(BP_*, BP_*(T(m))),$$

which is isomorphic to $\mathrm{Ext}_{\Gamma(m+1)}(BP_*, BP_*)$ by the change-of-rings isomorphism. So this object is computable using the chromatic spectral sequence introduced in [4]. Define comodules M_m^n by

$$M_m^n = v_{m+n}^{-1} BP_*/(p, \dots, v_{m-1}, v_m^\infty, \dots, v_{m+n-1}^\infty)$$

as usual. Then the chromatic E_1-term is

$$E_1^{s,t} = \mathrm{Ext}^t_{\Gamma(m+1)}(BP_*, M_0^s).$$

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