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Varieties with nonconstant Gauss fibers

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ABSTRACT. We construct a 4-dimensional projective variety whose general fibers of the Gauss map γ are one-parameter hyperplane sections of the given surface in $\mathbf{P}^3$ when the characteristic is positive. As an application, we have a projective variety whose general fibers of the Gauss map are not constant. In particular, this is a new example of a variety with non-linear Gauss fibers.

1. Introduction

The *Gauss map* γ on a projective variety $X \subset \mathbf{P}^N$ is the rational map from X to the Grassmannian $\mathbf{G}(\dim X, N)$ which assigns to a smooth point $p \in X$ the projective embedded tangent space $\mathbf{T}_p X$.

It is classically known that, if the characteristic of the base field K is 0, the general fiber of the Gauss map is a linear space (see, for example, [8]). In positive characteristic case, this is no longer true. There exist a curve which has infinitely many multiple tangent lines, hence the fibers of the Gauss map of this curve contain two distinct points ([7]). (A multiple tangent line is a line which has two or more distinct tangent points.) H. Kaji ([3], [4]), J. Rathmann ([6]) and A. Noma ([5]) found smooth varieties whose general fiber of the Gauss map has finitely many distinct points. By a result of F. L. Zak, the Gauss map on a smooth variety is a finite map onto its image ([8, I. 2.8]). Recently, the author found (singular) varieties whose general Gauss fiber is *not* a finite union of linear subspaces ([1]). More strongly, he proved that any given projective variety Y is (the reduced structure of) the general fiber of the Gauss map on some variety X ([2]). Note that in this construction, all the general fibers (with reduced structures) are isomorphic to each other.

In this paper we will construct first examples of "nonconstant" Gauss fiber structures. More concretely, when $Y \subset \mathbf{P}^3$ is a generic surface, we construct a 4-dimensional variety X such that the Gauss fibers are one-parameter hyperplane sections of Y.

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