Function of Self-Adjoint Transformation.

By

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(Received April 20, 1936.)

Introduction.

Let H be a self-adjoint transformation in Hilbert space $\mathfrak{H}$, $E(U)$ be the corresponding resolution of the identity in the sense of F. Maeda[1] which depends on Borel set U of 1-dimensional Euclidean space R_1, and $f(\lambda)$ be any Baire function defined in R_1. If we put

$$F(H) = \int_{R_1} f(\lambda) E(dU),$$

then $F(H)$ is a closed linear transformation,[2] which we call a function of H.

J. v. Neumann[3] and F. Riesz[4] have shown the theorem equivalent to the following: *When H is a bounded self-adjoint transformation, then a bounded linear transformation A is a function of H, if and only if A is permutable with any bounded linear transformation which is permutable with H.* And F. Riesz has remarked that anologous theorems hold for the unbounded cases.

In this paper we shall show that any transformation A defined on a subset of $\mathfrak{H}$ is a contraction of a function of H when A is permutable with any bounded linear transformation[5] permutable with H.[6] Moreover A is a function of H if and only if A is a closed linear trans-

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(1) F. Maeda, this journal, **4** (1934), 78.

(2) F. Maeda, *ibid.* 85–88.

(3) J. v. Neumann, Math. Ann. **102** (1929), 370–427. Annals of Math., **32** (1931), 191–226.

(4) F. Riesz, Acta Szeged, **7** (1935), 147–159.

(5) In the following proofs it is sufficient to consider only bounded linear transformations with norm $\leqq 1$.

(6) Prof. Y. Mimura of Osaka University reported a similar result in the annual meeting of the Physico-Mathematical Society held at Tokyo on April, 2, 1936.