Space of Differential Set Functions.

By

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In my previous papers,(1) I have investigated the space of set functions $\mathfrak{L}_2(\beta)$. It is defined as follows: Let $\mathfrak{K}$ be a closed family (σ-Körper) of all Borel subsets of a separable metric space Ω, and $\beta(E)$ be a completely additive, non-negative set function defined in $\mathfrak{K}$. If $\phi(E)$ be a completely additive set function defined in $\mathfrak{K}$, which is absolutely continuous with respect to $\beta(E)$ and $\int_\Omega | D_{\beta(E)}\phi(a) |^2 d\beta(E)$ is finite, then I said that $\phi(E)$ belongs to $\mathfrak{L}_2(\beta)$. $\mathfrak{L}_2(\beta)$ is a Hilbert space with the inner product

$$(\phi, \psi) = \int_\Omega D_{\beta(E)}\phi(a)\overline{D_{\beta(E)}\psi(a)}d\beta(E) .^{(2)}$$

In these previous papers, I have assumed that $\beta(\Omega)$ is finite. But in the applications, the case often occurs where $\beta(\Omega)$ is infinite. In this case, the usual definition of an integral is inconvenient. But A. Kolmogoroff(3) gave a new definition of an integral which is irrespective of the finiteness of $\beta(\Omega)$. In his definition of an integral, it is unnecessary that set functions are defined for all sets in a closed family; they need only be defined for decomposed sets of a multiplicative system. Such set functions, I call, in this paper, differential set functions. Using Kolmogoroff's integral, we can define the space of differential set functions in the same way as the space of ordinary set functions.

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(1) F. Maeda, "On the Space of Real Set Functions," this journal, **3** (1933), 1–42; "On Kernels and Spectra of Bounded Linear Transformations," ibid., 243–273; "Kernels of Transformations in the Space of Set Functions," this journal, **5** (1935), 107–116; "Transitivities of Conservative Mechanism," this volume, 1–18.

(2) If we do not demand the separability of $\mathfrak{L}_2(\beta)$, we can, more generally, take the closed family $\mathfrak{K}$ in an abstract space Ω as the domain of definition of set functions.

(3) A. Kolmogoroff, "Untersuchungen über den Integralbegriff," Math. Ann. **103** (1930), 654–682.