Kernels of Transformations in the Space of Set Functions.

By

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(Received April 20, 1935.)

Let $\beta(E)$ be a completely additive, non-negative set function defined for all Borel subsets of a Borel set A in a separable metric space. Let $\phi(E)$ be a complex valued set function which is absolutely continuous with respect to $\beta(E)$. When $\int_A |D_{\beta(E)}\phi(a)|^2 d\beta(E)$ is finite, $\phi(E)$ is said to belong to the class $\mathfrak{L}_2(\beta)$. Then $\mathfrak{L}_2(\beta)$ is a Hilbert space with the inner product.

$$(\phi, \psi) = \int_A D_{\beta(E)}\phi(a)\overline{D_{\beta(E)}\psi(a)}d\beta(E)\,.^{(1)}$$

In a previous paper,[(2)] I proved that all bounded linear transformations T defined in $\mathfrak{L}_2(\beta)$ can be expressed in the integral form

$$T\phi(E) = \int_A D_{\beta(E')}\mathfrak{K}(E, a')D_{\beta(E')}\phi(a')d\beta(E')\,, \qquad (1)$$

and the kernels of T are expressed as follows:

$$\mathfrak{K}(E, E')\,[=]_{E,E'} \sum_\nu \zeta_\nu(E)\overline{\psi_\nu(E')}\,,^{(3)}$$

where $\{\psi_\nu(E)\}$ is a complete normalized orthogonal system in $\mathfrak{L}_2(\beta)$ and

$$\zeta_\nu(E) = T\psi_\nu(E) \qquad (\nu = 1, 2, \ldots\ldots).$$

Of course, $\mathfrak{K}(E, E')$ belongs to $\mathfrak{L}_2(\beta)$ as a function of set E and of set E'. In this case, I say that $\mathfrak{K}(E, E')$ belongs to $\mathfrak{L}_2(\beta, \beta)$.

(1) Cf. F. Maeda, this journal, **3** (1933), 243; and **4** (1934), 141–142.

(2) F. Maeda, this journal, **3** (1933), 244–251.

(3) This expression means that $\sum_\nu \zeta_\nu(E)\overline{\psi_\nu(E')}$ converges strongly to $\mathfrak{K}(E, E')$ as functions of set E and of set E'.