# An Extension of Parallel Displacement by Matrices.

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## I.

Let us suppose that $A(t) = ((a_{ij}(t)))$ is a $L$-integrable matrix of $t$ in an interval $(p \leqq t \leqq r)$, and $Y^0$ is any matrix.[1] Here the order of the product of matrices is read from right to left.[2] If we put

$$Y^{\nu} = \prod_{k=1}^{\nu} \left\{ I + (t_k - t_{k-1}) A(\xi_{k-1}) \right\} Y^0 \quad \text{and} \quad t_{k-1} \leqq \xi_{k-1} < t_k ,$$

we have

$$Y = \lim_{m \to \infty} Y^m = \widehat{\int_p^r} (I + A(t)dt) Y^0 = \lim_{m \to \infty} \prod_{k=1}^{m} e^{(t_k - t_{k-1}) A(\xi_{k-1})} Y^0 . \quad (1)$$

The above defined $Y$ is called the product integral of $Y^0$ with respect to the matrix $A(t)$ from $t = p$ to $r$. Then we have as Caque-Fuchs's expansion of $Y$

$$\widehat{\int_p^r} (I + A(t)dt) Y^0 = \left\{ I + \int_p^r (A(t) \int_p^t A(t_1)dt_1)dt + \ldots . \right\} Y^0 . \quad (2)$$

Now we shall express the product integral of matrices in the form of an expansion using differential operations instead of the integral operations shown in (2). Form (1),

$$Y = \lim_{m \to \infty} \left\{ (I + A(t_{m-1})\Delta t) \ldots . (I + A(t_2)\Delta t)(I + A(t_1)\Delta t)(I + A(t_0)\Delta t) \right\}$$

(1) L. Schlesinger, *Math. Zeits.* **33** (1931), 33-61; **35** (1932), 485-501.

(2) This is the opposite of the usual order.