Theory of Vector Valued Set Functions.

By

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Let $\mathfrak{H}$ be an abstract Hilbert space,[1] and $\{\mathfrak{f}_\nu\}$ be a sequence of elements of $\mathfrak{H}$. If there exists an element $\mathfrak{f}$ in $\mathfrak{H}$ such that

$$\lim_{\nu\to\infty} \|\mathfrak{f}_\nu - \mathfrak{f}\| = 0\,,$$

then I say that $\{\mathfrak{f}_\nu\}$ converges strongly to $\mathfrak{f}$, and I write thus

$$[\lim_{\nu\to\infty}]\, \mathfrak{f}_\nu = \mathfrak{f}\,.$$

If a series of elements

$$a_1\mathfrak{f}_1 + a_2\mathfrak{f}_2 + \ldots\ldots + a_\nu\mathfrak{f}_\nu + \ldots\ldots \tag{1}$$

be such that $$[\lim_{\nu\to\infty}]\, \mathfrak{s}_\nu = \mathfrak{f}$$

where $$\mathfrak{s}_\nu = a_1\mathfrak{f}_1 + a_2\mathfrak{f}_2 + \ldots\ldots + a_\nu\mathfrak{f}_\nu\,,$$

then I say that the series (1) converges strongly to $\mathfrak{f}$, and I write as follows

$$\mathfrak{f}\,[=]\sum_\nu a_\nu\mathfrak{f}_\nu\,.$$

In the strongly convergent series, the most important is the expansion of any element $\mathfrak{f}$ with respect to a complete normalized orthogonal system $\{\mathfrak{g}_\nu\}$ in $\mathfrak{H}$:

$$\mathfrak{f}\,[=]\sum_\nu (\mathfrak{f}, \mathfrak{g}_\nu)\,\mathfrak{g}_\nu\,. \tag{2}$$

(1) For the abstract Hilbert space, cf. J. v. Neumann, *Mathematische Grundlagen der Quantenmechanik*, (1932); and M. H. Stone, *Linear Transformations in Hilbert Space*, (1932).