On Certain Functional Inequalities.

By

Kiyoshi TODA.

(Received October 10, 1933)

I

Mr. Jensen[1] defined *a convex function* as a real one-valued function which satisfies the inequality

$$\frac{\varphi(x)+\varphi(y)}{2} \geqq \varphi\left(\frac{x+y}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

independently of the values of variables x and y in the given interval (α, β) and has proved that the inequality

$$\sum_{i=1}^{n} p_i\varphi(x_i)\Big/\sum_{i=1}^{n} p_i \geqq \varphi\left(\sum_{i=1}^{n} p_i x_i\Big/\sum_{i=1}^{n} p_i\right) \quad \ldots\ldots\ldots \quad (2)$$

occurs for any positive quantities p_i. Conversely I wish to show, in this paper, that if (2) holds good independently of the variables x_i for certain p_i's and n, then (1) also must hold, thus the inequality (2) takes place for and only for a convex function; in other words, the solution of the given functional inequality (2) is convex.

If the sum of some p_i's ($p_i+p_j+\ldots\ldots+p_k = p$ say) is half the total sum $\sum_{i=1}^{n} p_i$, then by putting

$$x_i = x_j = \ldots\ldots = x_k = x \text{ and the remaining } x\text{'s} = y$$

in (2), we clearly obtain (1). Even if this assumption does not hold, we can solve (2) by putting

$$x_1=x,\ x_2=x_3=\ldots\ldots=x_n=y \quad \text{and} \quad p_1=p,\ p_2+p_3+\ldots\ldots p_n=q,$$

(1) Acta Math. (1906).