# Indices of the Orthogonal Systems in the Non-Separable Hilbert Space.

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Let $\mathfrak{H}$ be an abstract Hilbert space; that is, $\mathfrak{H}$ is a linear vector space where the inner product is defined, and it is complete. When $\mathfrak{H}$ is separable, we usually use the natural number for the index of the orthogonal system of elements in $\mathfrak{H}$, i. e. $\{\mathfrak{g}_n\}$, $n$ being natural numbers, and

$$(\mathfrak{g}_m, \mathfrak{g}_n) = \delta_{mn} ,$$

where $\delta_{mn} = 0$ when $m \neq n$, and $= 1$ when $m = n$. When $\{\mathfrak{g}_n\}$ is complete in $\mathfrak{H}$, $\{\mathfrak{g}_n\}$ is used as a basic of the representation of $\mathfrak{H}$. Put

$$a_n = (\mathfrak{f}, \mathfrak{g}_n) ,$$

then the sequence $\{a_n\}$ of complex numbers is the representative of $\mathfrak{f}$.

It is convenient in the theory of quantum mechanics to use a representative in which the elements of the basic system $\{\mathfrak{g}_n\}$ are eigenvectors of a self-adjoint operator. But the orthogonal system $\{\mathfrak{g}_n\}$ whose index is the natural number is applicable only in the case when the chosen self-adjoint operator has the discrete spectrum. When the chosen self-adjoint operator has the continuous spectrum, its eigenvectors have as index the real number, so that it is expressed by $\{\mathfrak{g}_r\}$ and it satisfies the following condition

$$(\mathfrak{g}_r, \mathfrak{g}_s) = \delta(r-s) ,$$

where $\delta(x)$ is the Dirac improper $\delta$ function, defined by

$$\int_{-\infty}^{\infty} \delta(x)dx = 1$$

$$\delta(x) = 0 \qquad \text{for } x \neq 0 .$$

In this case the representative of any element $\mathfrak{f}$ is a point function