Domains of Representatives of Linear Operators.

By

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(Received Sept. 20, 1936.)

In a previous paper,[1] one of the authors investigated the representatives of linear operators. Let $\mathfrak{H}$ be a complete linear vector space, where the inner product is defined, and let $\mathfrak{q}(U)$ be a completely additive vector valued differential set function, such that $\{\mathfrak{q}(U)\}$ is complete in $\mathfrak{H}$. Taking $\mathfrak{q}(U)$ as the basic of representation, we can represent $\mathfrak{H}$ by the space of differential set functions $\mathfrak{L}_2(\sigma)$, where $\sigma(U) = \| \mathfrak{q}(U) \|^2$. Let $\boldsymbol{T}$ be a linear operator in $\mathfrak{H}$ which transforms $\mathfrak{f}$ to $\mathfrak{g}$. That is

$$\mathfrak{g} = \boldsymbol{T}\mathfrak{f}.$$

Corresponding to this operator, we have an operator in $\mathfrak{L}_2(\sigma)$, which transforms the representative $\xi(U)$ of $\mathfrak{f}$ to the representative $\eta(U)$ of $\mathfrak{g}$. We may denote it by the same symbol $\boldsymbol{T}$ so that

$$\eta(U) = \boldsymbol{T}\xi(U).$$

On the other hand, let $\mathfrak{K}(U, U') = \big(\boldsymbol{T}\mathfrak{q}(U'), \mathfrak{q}(U)\big)$ be the representative of $\boldsymbol{T}$. Then we have

$$\eta(U) = \int_V \frac{\mathfrak{K}(U, dU')\, \xi(dU')}{\sigma(dU')}.$$

But the last integral is an integral operator in $\mathfrak{L}_2(\sigma)$, which we denote by $\boldsymbol{T}_{\mathfrak{K}}$, so that

$$\eta(U) = \boldsymbol{T}_{\mathfrak{K}}\xi(U).$$

Thus we have the relation:

$$\boldsymbol{T} \subseteqq \boldsymbol{T}_{\mathfrak{K}}.$$ [2]

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(1) F. Maeda, "Representations of Linear Operators by Differential Set Functions," this journal, **6** (1936), 115–137.

(2) This means that $\boldsymbol{T}_{\mathfrak{K}}$ is an extension of $\boldsymbol{T}$.