Relation between Intuitionistic Logic and Lattice.

By

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A. Tarski[(1)] has recently obtained interesting results about the relations between the propositional calculi and topologies, which lead to the question: What are the propositional calculi in lattice terms? The classical propositional calculus is the Boolean Algebra. Here I shall show that the intuitionistic propositional calculus is a residuated lattice closed with respect to the lattice operation, meet, which has a null element. For convenience we refer to A. Tarski's set of postulates of propositional calculi, and the proof shall be effected by characterizing the implication and negation in lattice terms.

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§ 1. Let $\rightarrow$, $\wedge$, $\vee$, —, be the four fundamental operators in the propositional calculi, or logical constants, the first three of which are binary operators, but the last is unary. Let A, B, C, ..., be expressions formulated from the propositional variables and the above four operators. Following A. Tarski we shall here reproduce the postulates of the propositional calculi:[(1)]

(i) $A \rightarrow (B \rightarrow A)$.

(ii) $[A \rightarrow (B \rightarrow C)] \rightarrow [(A \rightarrow B) \rightarrow (A \rightarrow C)]$.

(iii) $(A \wedge B) \rightarrow A$.

(iv) $(A \wedge B) \rightarrow B$.

(v) $(C \rightarrow A) \rightarrow \{(C \rightarrow B) \rightarrow [C \rightarrow (A \wedge B)]\}$.

(vi) $A \rightarrow (A \vee B)$.

(vii) $B \rightarrow (A \vee B)$.

(viii) $(A \rightarrow C) \rightarrow \{(B \rightarrow C) \rightarrow [(A \vee B) \rightarrow C]\}$.

(1) A. Tarski, Fundamenta Math. **31** (1939), 103–134.