# Lattice Functions and Lattice Structure.

By

Fumitomo MAEDA.

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G. Birkhoff(1) has proved that if in a lattice $L$ a dimension function is defined, then $L$ is a modular lattice. And J. v. Neumann(2) says that if in a complemented continuous lattice $L$ a unique dimension function which has some particular properties is defined, then $L$ is a continuous geometry. These are remarkable facts, which show that the dimension function restricts the structure of the lattice.

In the present paper I investigate this problem in a general way. Let $L$ be a lattice, and a real valued function $\phi(a)$ be defined for all $a \in L$; thus we may say that $\phi(a)$ is a lattice function. The properties of this lattice function $\phi(a)$ may be given in the following way:

(i) $\phi(a)$ is additive when

$$\phi(a \cup b)+\phi(a \cap b)=\phi(a)+\phi(b).$$

(ii) $\phi(a)$ is completely additive when

$$\phi\left(\sum(a_i; i=1, 2, \ldots.)\right)=\sum_i \phi(a_i)$$

for any independent system $(a_i; i=1, 2, \ldots.)$.

(iii) $\phi(a)$ is non-decreasing when

$$a<b \quad \text{implies} \quad \phi(a) \leqq \phi(b).$$

(iv) $\phi(a)$ is increasing when

$$a<b \quad \text{implies} \quad \phi(a)<\phi(b).$$

I first investigate the relations between the increasing and the non-decreasing properties of the additive function $\phi(a)$ and the struc-

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(1) G. Birkhoff [1], 744. The numbers in square brackets refer to the list given at the end of this paper.

(2) J. v. Neumann [1], 99.