Prime Ideals in Boolean Rings.

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1. Introduction. Boolean rings may be regarded as abstract commutative rings in which every element is idempotent.(1) The ideal theory of these rings can be developed easily as a special instance of the ideal theory of general rings,(2) and the important results of the algebraic theory of Boolean rings developed by M. H. Stone(3) will follow from the general ideal theory. In the present paper we shall be concerned primarily with the properties of the prime ideals(4) in Boolean rings which involve the *well-ordering hypothesis.*

The following lemma has been found valid for idempotent principal ideals in general commutative rings,(5) and Stone(6) has proved it in Boolean rings. We shall prove it here merely for its importance for further investigation.

Lemma. *If* $\mathfrak{a}(\neq \mathfrak{o})$ *is a principal ideal in a Boolean ring* A, A *can be represented as the direct sum* $\mathfrak{a}+\mathfrak{a}'$, *where* $\mathfrak{a}'$ *is the orthocomplement of* $\mathfrak{a}$.

From the definition of a principal ideal, it is evident that $\mathfrak{a}$ is the subclass consisting of all the elements b such that, for a fixed non-zero element a, $ab=b$. If we denote the subclass consisting of all the elements c orthogonal to the element a by $\mathfrak{a}'$, then $\mathfrak{a}'$ is an ideal orthogonal to $\mathfrak{a}$. We let d be an arbitrary element in A and write

$$d=ad+d-ad\,.$$

(1) M. H. Stone, The Theory of Representations for Boolean Algebras, Trans. of Amer. Math. Soc. **40** (1936), 39.

(2) W. Krull, Idealtheorie in Ringen ohne Endlichkeitsbedingung, Math. Annalen **101** (1929), 729.

(3) Stone, loc. cit., 100–105.

(4) Ibid., 100. W. Krull, loc. cit., 735.

(5) S. Mori, Über allgemeine Multiplikationsringe, this Journal **4** (1934), I. *A* Boolean ring is a special multiplicative ring.

(6) Stone, loc. cit., 63.