Ring-Decomposition without Chain-Condition.

By

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In algebra, the decomposition of a ring in a direct sum of simple right ideals is discussed on the basis of "chain-condition" or "minimum-condition." Thus, a ring $\mathfrak{R}$ without radical, with minimum-condition for right ideals, is a direct sum of simple right ideals, i. e.

$$\mathfrak{R}=\mathfrak{a}_1+\mathfrak{a}_2+\cdots\cdots+\mathfrak{a}_n\,, \tag{1}$$

and there exist idempotents e_i $(i=1, 2, \ldots\ldots, n)$, such that

$$\mathfrak{a}_i=(e_i)_r\,, \qquad e_ie_j=0 \quad \text{when} \quad i\neq j\,,$$

and

$$1=e_1+e_2+\cdots\cdots+e_n\,.^{(1)}$$

When the ring $\mathfrak{R}$ does not satisfy the minimum-condition, we cannot decompose $\mathfrak{R}$ in a direct sum of *simple* right ideals as in (1). Hence we must consider ring-decomposition from another point of view. Since the set $R_{\mathfrak{R}}$ of all right ideals is a lattice,[2] from the point of view of the lattice theory we can investigate the set of right ideals which are used for the decompositions of $\mathfrak{R}$.

For example, consider the case where $\mathfrak{R}$ without radical satisfies the minimum-condition. Then the decomposition (1) shows that $\mathfrak{R}$ is the join of right ideals $(\mathfrak{a}_1, \mathfrak{a}_2, \ldots\ldots, \mathfrak{a}_n)$. Let V be the set of n positive integers $1, 2, \ldots\ldots, n$; and let U be any subset of V, whose elements are $i_1, i_2, \ldots\ldots, i_\nu$. And write

$$\mathfrak{a}_U=\mathfrak{a}_{i_1}+\mathfrak{a}_{i_2}+\cdots\cdots+\mathfrak{a}_{i_\nu}\,.$$

Then

$$\mathfrak{a}_{U_1}\frown\mathfrak{a}_{U_2}=(0) \qquad \text{when} \qquad U_1U_2=0\,.$$

And when V is a sum of mutually disjoint sets, i. e.

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(1) B. L. van der Waerden [1], 156–161. The numbers in square brackets refer to the list given at the end of this paper.

(2) J. v. Neumann [5], 4.