A Generalization of Rumer's Form of Maxwell's Equation in Riemannian Space.

By

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1. Maxwell's electromagnetic equation in vacuum is given by

$$
(1.1)\quad \begin{cases} \operatorname{div} \mathfrak{H}=0\,, \qquad \operatorname{div} \mathfrak{E}=0 \\ \operatorname{rot} \mathfrak{E}+\dfrac{1}{c}\dfrac{\partial \mathfrak{H}}{\partial t}=0\,, \qquad \operatorname{rot} \mathfrak{H}-\dfrac{1}{c}\dfrac{\partial \mathfrak{E}}{\partial t}=0\,. \end{cases}
$$

In the theory of Relativity, Maxwell's equation in the form (1.1) is generalized in the form of the following tensor equations[1]:

$$
(1.2)\quad \begin{cases} F_{ij}=\dfrac{\partial \varphi_i}{\partial x^j}-\dfrac{\partial \varphi_j}{\partial x^i} \\ \nabla_j F^{ij}=0 \qquad (i,j=1,2,3,4)\,, \end{cases}
$$

where ∇_j denotes the Riemannian covariant derivative, and $\varphi^i=g^{ij}\varphi_j$ is the contravariant vector of a potential whose first three components and fourth component coincide with the vector and the scalar potentials, respectively, in the Galilean coordinate system.

On the other hand, Maxwell's equation (1.1) can be written in a complex form as follows

$$
(1.3)\quad \begin{cases} \operatorname{div}(\mathfrak{H}+i\mathfrak{E})=0 \\ \operatorname{rot}(\mathfrak{H}+i\mathfrak{E})-\dfrac{\partial}{ic\partial t}(\mathfrak{H}+i\mathfrak{E})=0 \end{cases}
$$

and G. Rumer[2] has rewritten this equation in the following matrix form:

$$
\begin{cases} \mathring{D}\mathfrak{F}=0 & (1.4) \\ F_4=0 & (1.4\text{a}) \end{cases}
$$

$$
(1.5)\quad \mathring{D}=\mathring{\sigma}^i\frac{\partial}{\partial x^i}\,, \qquad [x^1\equiv x,\ x^2\equiv y,\ x^3\equiv z,\ x^4\equiv -ict]
$$

(1) A. S. Eddington, The Mathematical Theory of Relativity, Cambridge. 2nd ed. (1930), 173.

(2) G. Rumer, Zs. f. Physik, **65** (1930), 244.