Wave Geometry No. 45

Mathematical Foundation of Wave Geometry. II.
A Generalization of Clifford Number.

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Summary.

In this continuation of an earlier paper[1] we shall investigate some properties of the iteration system.

In § 6, it will be proved that if an iteration system $\mathfrak{B}$ is M-divisible, there exists the unit element of $\mathfrak{B}$, and conversely; and the condition of h's being the unit element of $\mathfrak{B}$ is $h(M)\cdot M=M$.

In § 7, we shall investigate the relations between the numbers q and p (the degree and the order) of linearly independent basic elements of $\mathfrak{B}$ with the reference fields of $\{\mathfrak{K}, M\}$ and $\mathfrak{K}$ respectively, and the closed dimension n of $\mathfrak{B}$.

In § 8, we shall obtain the matrix-representation of $\mathfrak{B}$, and prove that the iteration systems are essentially classified in the two types.

In § 9, we shall investigate the relations between any two sets of the central bases a_i and a'_i $(i=1,\ldots,n)$ in an iteration system.

§ 6. Unit element.

We put forward the following two definitions.

Definition: When we can always conclude that, for any element $\beta_0\in\mathfrak{B}$, $\beta_0=0$ from $M\beta_0=0$, $\mathfrak{B}$ is called *M-divisible.*

Definition: h is called the *unit element* of $\mathfrak{B}$ when the element h of $\mathfrak{B}$ satisfies the following relations for any element β of $\mathfrak{B}$;

$$h\beta=\beta, \qquad \beta h=\beta.$$

Then we have the following theorems concerning the unit element of $\mathfrak{B}$.

Theorem 21. *When $\mathfrak{B}$ is M-divisible, the necessary and sufficient condition for h's being the unit element of $\mathfrak{B}$ is that*

$$Mh=M$$

and as the definition for the unit element one of the relations $h\beta=\beta$ and $\beta h=\beta$ may be omitted.

(1) K. Morinaga, this Journal, **10** (1940), 215 (W.G. No. 40.) (We refer to this as **I.**)