# Mathematical Foundations of Wave Geometry. I.

By

Kakutaro MORINAGA.

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## Introduction and Summary.

Wave Geometry is constructed by taking $ds\psi=\gamma_i dx^i\psi\,(\gamma_{(i}\gamma_{j)}=g_{ij}I)$ as the metric of 4-dimensional manifold. This metrical form is introduced to unify general relativity and quantum mechanics in a natural way from the physical angle; but the mathematical basis for adopting such a metric representation has not yet been fully established. Hence to establish the foundations of Wave Geometry, it is very important to pursue the problem along this line.

The purpose of this paper is (1) to construct a theory of number system $\mathfrak{B}$ (or operator system) which shall be a generalization and abstraction of Dirac's $\gamma_i$ or Eddington's E-number, (2) to show how this new system can imply, as a special case, Cliford's number, including Dirac's matrices $\gamma_i$ and Eddington's E-numbers, and how it is related to matrix representation, and (3) to establish the methematical foundations of Wave Geometry by constructing a geometry of the manifold of this system, without using coordinates independent of number system or operator as in the geometry of operators hitherto proposed.

This investigation especially distinguished from those in Wave Geometry hitherto have previously appeared in the following respects: Elements of the ideal in this number system $\mathfrak{B}$ play the rôle of $\psi$ (wave function) used as operands for operators; therefore all quantities in consideration belong to $\mathfrak{B}$. And, using this ideal, we introduce the conception of "norm" of any element in $\mathfrak{B}$, by which we can discuss convergency of any series in $\mathfrak{B}$.

In this paper, as the first step, we shall restrict our investigation to when the number of linearly independent elements in $\mathfrak{A}$ is finite, leaving the case of infinite linearly independent elements for future papers now in preparation.

In § 1, we construct from a general (original) operator set $\mathfrak{M}$ and a corpus $\mathfrak{K}$ a linear manifold $\mathfrak{A}$ in the field $\mathfrak{K}$ satisfying Axiom I. Then we find some relations among $\mathfrak{M}$, $\mathfrak{K}$, and $\mathfrak{A}$ (Theorem 1).

In § 2, we introduce the multiplication and addition of elements of $\mathfrak{A}$, and construct a manifold $\mathfrak{B}$ from the multiplication and addition of ele-