Wave Geometry No. 39

Cosmology and Conformally Flat Space.

By

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(Received May 18, 1940.)

§ 1. Introduction.

At present there exist three cosmological theories, namely, relativistic cosmology, kinematical cosmology, and cosmology in terms of Wave Geometry. Though the fundamental ideas of these three theories differ radically from one another, there are intimate connections among the line elements considered in them; moreover, almost all of them are related to those of conformally flat space.(1) It will therefore be worth while to investigate the relations between the spaces considered in the cosmological theories and the conformally flat spaces.

The purpose of this paper is

(A) to show that (1) there exist close connections between the line elements considered in cosmology in terms of Wave Geometry and those of conformally flat space, (2) the most general expression of the line element in relativistic cosmology(2)

$$ds^2=-\frac{e^{2g(t)}}{\left[1+\frac{r^2}{4R^2}\right]^2}(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2)+dt^2 \qquad (1.1)$$

is also that of conformally flat space and (3) the spaces adopted by kinematical cosmology generalized by Walker and by McVittie are involved in (1.1), and

(B) to deduce the several geometrical properties of the conformally flat spaces.

In §§ 2 and 3 we shall first establish the theorems characterizing conformally flat space and study, as their applications, the properties of de Sitter-type and Einstein-type universes. Next, in §§ 4 and 5, we shall obtain the condition for a universe with electromagnetism to be conformally flat and, making use of this result, study the properties of these spaces. We shall then investigate, in § 6, the general conformally flat space with

(1) Throughout this paper we use "*conformally flat space*" in place of "space which is conformal to a flat space."

(2) H. P. Robertson, Reviews of Mod. Phy., **5** (1933), 62. R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, (1934), 369.