EQUATIONS OF SCHRÖDER (Continued)

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Chapter III. Correspondence of the eigen values.

§ 1. Preliminaries.

In Chap. II, we have noticed that, when (2.12) holds, (2.14) always holds, however, when (2.14) holds, (2.12) does not necessarily hold. In this chapter, we study the condition that, for the given values of λ_i, there exists a set of values μ_i such that, for any sets of $(p_1, p_2, \dots, p_R)$ satisfying Chap. II (2.14), there always hold Chap. II (2.12), namely Chap. II (2.12) and (2.14) are in one-to-one correspondence.

We assume that either $0<|\lambda_i|<1$ or $|\lambda_i|>1$. Then, by the suitable arrangement of λ_i, as seen from Chap. I § 1, the relations Chap. II (2.14) which really hold are of the forms as follows:

$$\lambda = \lambda_1^{p_1}\lambda_2^{p_2}\dots\dots\lambda_N^{p_N} \tag{1.1}$$

where λ is an eigen value λ_i such that $i>N$. We denote by $\mathring{\mu}_i$ arbitrary value of μ_i determined by Chap. II (2.8) for given λ_i. Then, for one set $(\mathring{p}_1, \mathring{p}_2, \dots, \mathring{p}_N)$ satisfying (1.1),

$$\sum_{i=1}^{N} \mathring{\mu}_i \mathring{p}_i = \mathring{\mu} \ , \tag{1.2}$$

where $\mathring{\mu}$ is a suitably determined value of μ corresponding to λ. For any set of $(p_1, p_2, \dots, p_N)$ satisfying (1.1),

$$\sum_{i=1}^{N} \mathring{\mu}_i p_i = \mathring{\mu} + n\frac{2\pi}{t_0}\sqrt{-1} \ , \tag{1.3}$$

where n is an integer corresponding to the set of $(p_1, p_2, \dots, p_N)$. Put $\mathring{\mu}_i = A^i + B^i\sqrt{-1}$ and $\mathring{\mu} = A + B\sqrt{-1}$. Then, making use of the convention of tensor calculus, it follows that

$$(p_i - \mathring{p}_i)A^i = 0 \ , \quad (p_i - \mathring{p}_i)B^i = \frac{2\pi}{t_0}n \ . \tag{1.4}$$

The sets of $(p_1, p_2, \dots, p_N)$ satisfying (1.1) are determined by (1.4). Then, for all such sets of $(p_1, p_2, \dots, p_N)$, we seek for n^i and m such that $\sum_{i=1}^{N}(\mu_i + \frac{2\pi}{t_0}n^i\sqrt{-1})p_i = \mathring{\mu} + m\frac{2\pi}{t_0}\sqrt{-1}$, i.e.