EQUATIONS OF SCHRÖDER

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Preface.

By means of the iteration method, Schröder[1] has solved the functional equation as follows: $f[\varphi(x)]=\lambda f(x)$, where $\varphi(x)$ is a given regular function such that the expansion of $\varphi(x)$ is as follows: $\varphi(x)=\lambda x+\cdots$, and $|\lambda|>1$ or $0<|\lambda|<1$. In 1945, Fukuhara[2] has extended the equation into those of many variables. For the given functions $\varphi^\mu(x)$ such that $\varphi^\mu(x)=a_\nu^\mu x^\nu+\cdots$, (det.$|a_\nu^\mu|\neq0$), he has considered the functional equations as follows:

$$f_i[\varphi(x)] = \lambda_i f_i(x)+\delta f_{i-1}(x)+\Psi_i(x) ,$$

where λ_i are the eigen values of $\|a_\nu^\mu\|$ and δ is an arbitrary number such that $|\delta|$ is sufficiently small and $\Psi_i(x)$ are the suitable polynomials of the functions f_i. Under the condition that the absolute values of all the eigen values of $\|a_\nu^\mu\|$ are less or greater than unity, he has solved the equations.

In this paper, we study the extended equations of Schröder from different point of view.

In Chapter I, we add some remarks on Fukuhara's paper. In Chapter II, we consider the transformation $\mathfrak{X}:\ 'x^\mu=\varphi^\mu(x)$. Assuming the existence of one parameter group of the transformations containing $\mathfrak{X}$, we show, that the equations of Schröder resemble closely to the finite forms of the characteristic equations[3] of a linear homogeneous partial differential equation. Thus, when some more conditions are satisfied, by making use of the results of the previous paper,[4] we easily get the solutions of the equations newly obtained. In Chapter III, we study the fundamental theorem which plays an important roll for the subsequent discussions. In Chapter IV, we consider the conditions assumed in Chap. II, and we find that, when the absolute values of all the eigen values of $\|a_\nu^\mu\|$ are less or greater than unity, there exists a one parameter group containing the

1) Schröder, Math. Ann., 1871.
2) Fukuhara, Kyūshū-Teikoku-Daigaku Rigaku-Hōkoku, Vol. 1, No.2 (1945).
3) Urabe, This Journal, Vol. 15, No. 1 (p.25).
4) do.