

## LATTICE THEORETIC CHARACTERIZATION OF ABSTRACT GEOMETRIES

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Frink [1]<sup>1)</sup> and Prenowitz [1] have characterized the lattice of linear subspaces of a projective geometry, and the lattice of convex sets of a descriptive geometry, all these geometries being of arbitrary dimensions, finite or infinite.

The purpose of this paper is to find the fundamental theorems which enable us to characterize all types of these geometries simultaneously. These theorems are as follows:

**THEOREM I.** *An abstract lattice  $L$  is isomorphic with the lattice of all subgeometries of a suitable abstract geometry  $G$  with finitary operations, if and only if it is a relatively atomic, upper continuous lattice.*

**THEOREM II.** *A relatively atomic, upper continuous lattice  $L$  is a direct sum of sublattices  $S_\alpha$  ( $\alpha \in I$ ) of  $L$ . And any two points in the same  $S_\alpha$  are connected, and two points which are contained in different  $S_\alpha$  and  $S_\beta$  are not connected.*

Thus there exists a one-one correspondence between a relatively atomic, upper continuous lattice  $L$  and an abstract geometry  $G$  with finitary operations. For example, when  $L$  is a relatively atomic, upper continuous distributive lattice, that is,  $L$  is an atomic, complete Boolean algebra, then the associated geometry  $G$  is a point set and subgeometries mean the subsets of  $G$ . When  $L$  is a relatively atomic, upper continuous, modular lattice, that is,  $L$  is an atomic, upper continuous, complemented modular lattice, then the associated geometry  $G$  is a projective geometry. Similarly Prenowitz's [1] investigations are special cases of Theorem I<sup>2)</sup>. In the last part of this paper, I generalize the Prenowitz's [1] results, introducing the concept "linear lattice".

Since a lattice  $L$  with 0 is relatively atomic, if and only if each element of  $L$  is the join of its contained points, Theorem I may be stated as follows:

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- 1) The numbers in square brackets refer to the list of references at the end of the paper.
  - 2) Prenowitz [1] did not consider about Theorem II.