INDEPENDENCE OF QUADRATIC QUANTITIES IN A NORMAL SYSTEM

By

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In the study of independence of quadratic quantities in a normal system Craig-Sakamoto's lemma is a fundamental one (1). We give here its simple proof, the method of which will also enable us in a short way to characterize quadratic quantities distributed in χ^2-distributions (Theorem 1) and to obtain an analogous result for Wishart's distributions (Theorem 14). In Part I we investigate the independence of quadratic quantities in a normal system to obtain results related to Cochran's theorem, and in Part II we extend results so obtained to the case of Wishart's distributions.

Part I. χ^2-distributions and Cochran's theorem

§ 1. Craig-Sakamoto's lemma.

In the following, unless otherwise stated, $A, B, C, \ldots$ stand for real symmetric matrices of order $k \cdot k$.

Lemma 1. The following two conditions are equivalent:

(i) $$AB = 0$$

(ii) $$|E-sA|\,|E-tB| = |E-sA-tB|$$

for arbitrary real scalars s, t.

Proof. Since evidently (i) implies (ii), so we shall show the converse. Let $K=(E-sA)^{-1}=E+sA+s^2A^2+\ldots$, then we may write (ii) as

$$|E-tB| = |E-tKB|$$

or

$$\sum_{n=1}^{\infty}\frac{t^n}{n}\, trB^n = \sum_{n=1}^{\infty}\frac{t^n}{n}\, tr\,(KB)^n .$$

Comparing the coefficients of s^2t^2 on both sides of this equation, we have

$$tr\,(ABAB+2A^2B^2) = 0 .$$

(1) A. T. Craig [1]. H. Sakamoto [1]. Numbers in brackets refer to the list of references at the end of this paper.