# On the Logarithmic Functions of Matrices. II.
## (On Some Properties of Local Lie Groups)

By

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### § 1. Logarithmic functions of real matrices.

In the preceding paper[1] we have obtained the following results about the logarithmic functions of complex matrices. We consider the complex matrices of order $n$. Let $\mathfrak{M}$ be the totality of regular matrices, $\tilde{\mathfrak{M}}$ the totality of regular matrices whose all characteristic values are not negative, $\mathfrak{A}_{(a)}$ the totality of matrices whose different characteristic values $\mu_i$ have the imaginary part $I(\mu_i)$ such that $a-\pi \leqq I(\mu_i) < a+\pi$ ($a$ is any real number), $\tilde{\mathfrak{A}}_{(a)}$ the totality of matrices such that $a-\pi < I(\mu_i) < a+\pi$ ($a$ is any real number), and $\mathfrak{A}^*$ the totality of matrices such that $\mu_i \not\equiv \mu_j \pmod{2\pi\sqrt{-1}}$ for all different characteristic values $\mu_i$.[2] Then we have the following properties:

(1) There exisits in $\mathfrak{A}_{(a)}$ one and only one matrix such that $exp\, A = M$ for a given matrix $M \in \mathfrak{M}$.

(2) The exponential mapping $A \rightarrow exp\, A = M$ is a topological mapping from $\tilde{\mathfrak{A}}_{(a)}$ onto $\tilde{\mathfrak{M}}$

(3) Let $A, B \in \mathfrak{A}^*$. $AB=BA$ if and only if $exp\, A\, exp\, B = exp\, B\, exp\, A$.

(4) Let $A \in \mathfrak{A}^*$. $A=\begin{pmatrix} U & W \\ O & V \end{pmatrix}$ if and only if $exp\, A=\begin{pmatrix} H & L \\ O & K \end{pmatrix}$, where $U$ and $V$ are the matrices of the same order as $H$ and $K$ respectively.

In this section we shal consider the logarithmic functions of real matrices. We denote by $\mathfrak{M}_{real}$, $\tilde{\mathfrak{M}}_{real}$, $\mathfrak{A}_{(0)real}$, $\tilde{\mathfrak{A}}_{(0)real}$ and $\mathfrak{A}^*_{real}$ the totality of the real matrices belonging to $\mathfrak{M}$, $\tilde{\mathfrak{M}}$, $\mathfrak{A}_{(0)}$, $\tilde{\mathfrak{A}}_{(0)}$ and $\mathfrak{A}^*_{real}$ respectively. Then it is obvious that the above properties (3) and (4) hold for $\mathfrak{A}^*_{real}$.

Next we shall investigate the properties (1) and (2) in the case of real matrices.

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1) K. Morinaga and T. Nôno : On the Lograrithmic Functions of Matrices I, Journal of Science of the Hiroshima University, Ser, A, Vol. 14, No. 2, 1950.

2) $\mathfrak{A}^* = Log(\mathfrak{M})$ and $\mathfrak{A}^* \subset \mathfrak{A}_{(a)}$.