Embedding Theorem of Continuous Regular Rings

By

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Let L be a reducible continuous geometry, and Ω the set of all maximal neutral ideals J in L. Kawada-Matsushima-Higuchi [1][1] has proved that L is isomorphic to a sublattice of $\Pi(L/J; J\varepsilon\Omega)$, where L/J are irreducible continuous geometries. In this paper, I apply this result to a reducible continuous regular ring $\mathfrak{R}$, the set $\bar{R}_{\mathfrak{R}}$ of all principal right ideals of $\mathfrak{R}$ being a reducible continuous geometry. And I obtain an embedding theorem of $\mathfrak{R}$. (Cf. Theorem 3·2, below.)

§ 1. Dimension Functions of Reducible Continuous Geometries.

Let L be a continuous complemented modular lattice, i. e. a reducible continuous geomertry, and Z the center of L. Then Z is a complete Boolean algebra. Denote by Ω the set of all maximal ideals $\mathfrak{p}$ of Z. For any $z\varepsilon Z$, let $E(z)$ be the set of all maximal ideals which do not contain z. Using $(E(z); z\varepsilon Z)$ as an additive basis for the open sets of Ω, Ω is a totally-disconnected bicompact Hausdoff space. T. Iwamura [1] proved that for any $a\varepsilon L$, there is a continuous functions $D(a)=\delta(a,\mathfrak{p})$ defined in Ω, which has the following properties;

(1°) $0\leqq D(a)\leqq 1,\quad D(0)=0,\quad D(1)=1.$

(2°) $a>0$ implies $D(a)>0$.

(3°) when $z\varepsilon Z$, $\delta(z,\mathfrak{p})=0$ or 1, according as $z\varepsilon\mathfrak{p}$ or not.

(4°) $D(a\vee b)+D(a\wedge b)=D(a)+D(b)$.

(5°) $a\gtreqless b$ are equivalent to $D(a)\gtreqless D(b)$ respectively.

LEMMA 1·1, *For any $a\varepsilon L$, let a real number $m(a)$ be defined as follows:*

(α) $0\leqq m(a)\leqq 1,\quad m(0)=0,\quad m(1)=1,$

(β) $z\varepsilon Z$ *implies* $m(z)=0$ *or* 1,

(γ) $m(a\vee b)+m(a\wedge b)=m(a)+m(b)$.

Put $\mathfrak{p}=(z;\ m(z)=0,\ z\varepsilon Z)$, $J=(a;\ m(a)=0)$. Then $\mathfrak{p}$ is a maximal ideal in Z, and J is a maximal neutral ideal in L. And $a\varepsilon J$ when and only when $r_n(A_a,A_1)\varepsilon\mathfrak{p}$ $(n=1,2\ldots)$[2].

PROOF. Cf. Kawada-Matsushima-Higuchi [1].

THEOREM 1·1. *Let J be a maximal neutral ideal in L, and $\mathfrak{p}$ a maximal ideal in Z. Then*

(1°) $\mathfrak{p}(J)=(z;\ z\varepsilon J,\ z\varepsilon Z)$ *is a maximal ideal in Z,*

(2°) $J(\mathfrak{p})=(a;\ \delta(a,\mathfrak{p})=0)$ *is a maximal neutral ideal in L,*

(1) The numbers in square brackets refer to the list given at the end of this paper.
(2) For the definition of $r_n(A_a,A_1)$ cf. v. Neumann [1] III 30.