LATTICE THEORETIC CHARACTERIZATION OF GEOMETRIES SATISFYING "AXIOME DER VERKNÜPFUNG"

By

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In my previous paper [1],[1] I have characterized lattice-theoretically an affine geometry of arbitrary dimensions,[2] i. e. a geometry satisfying the Euclidean axiom of parallel lines and "Axiome der Verknüpfung" of D. Hilbert [1],[3] except for the restrictions on the dimensionality.

The purpose of this paper is to characterize lattice-theoretically a geometry satisfying "Axiome der Verknüpfung" alone. The main theorem is as follows:

THEOREM. *An abstract lattice L is isomorphic to the lattice of all subspaces of a space satisfying "Axiome der Verknüpfung" of D. Hilbert* [1], *except for the restrictions on the dimensionality, if and only if L is a strongly plane matroid lattice.*[4]

1. We shall begin by showing several preliminary lemmas.

DEFINITION 1. Let A be a set of points such that for any pair of distinct points p, q there exists a subset $p \vee q$ (called *line*), containing p, q and for any triple of points p, q, r, which are not on a line, there is a subset $p \vee q \vee r$ (called *plane*) containing p, q, r, which satisfy the following conditions:

A. 1. *Two distinct points on a line determine the line.*

A. 2. *Three non-collinear points on a plane determine the plane.*

A. 2'. *The line through two distinct points on a plane is contained in the plane.*

By a *subspace* of A, we mean a subset S such that if p, q are distinct points of S, then $p \vee q \subseteqq S$ and if p, q, r are non-collinear points of S, then

1) The numbers in square brackets refer to the list of the references at the end of the paper.

2) Cf. U. Uasaki [1], Definition 2.

3) Cf. Ibid. 3 and 20.

4) Cf. Definition 3, below.