JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 17, NO. 3, MAR., 1954

System of Differential Equations which are Equivalent to Dirac's Equation for Hydrogen Atom

By

Takashi SHIBATA

(Received October 31, 1953)

§ 1. Introduction.

Dirac's wave equation is

$$\{E+e\varphi+\beta E_0+\sum_{k=1}^{3}\alpha_k(cp_k+eA_k)\}\psi=0, \tag{1.1}$$

where

$A_k\,(k=1,2,3)$: vector potential, φ: scalar potential, E: energy, $E_0=m_0c^2$, $p_k=-i\hbar\dfrac{\partial}{\partial x^k}\,(k=1,2,3)$, $\hbar=\dfrac{h}{2\pi}$, $\alpha_k\,(k=1,2,3)$ and β are 4—4 matrices such as

$$\alpha_1=\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix}\qquad \alpha_2=\begin{bmatrix}0&0&0&-i\\0&0&i&0\\0&-i&0&0\\i&0&0&0\end{bmatrix}$$

$$\alpha_3=\begin{bmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{bmatrix}\qquad \beta=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}$$

which satisfy the relations:

$$\alpha_{(k}\alpha_{l)}=\delta_{kl}, \qquad \beta\beta=1, \qquad \alpha_k\beta+\beta\alpha_k=0, \qquad (k, l=1,2,3)$$

ψ: 1—4 matrix having components $\psi_1, \psi_2, \psi_3, \psi_4$.

Denoting the space and time coordinates x, y, z and t by x^1, x^2, x^3 and x^4, the equation (1.1) can be expressed in the form which is symmetrical with respect to space and time coordinates as follows:

$$\gamma^i\left(\frac{\partial}{\partial x^i}-\varphi_i\right)\psi=\mu\psi \qquad (i=1,\cdots,4). \tag{1.2}$$

In this expression, according to usual convention which will be used throughout, the term of the left hand side stands for the sum of 4 terms as i take the values