Theory of the Spherically Symmetric Space-Times. VII. Space-Times with Corresponding Geodesics[1)]

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§ 1. Introduction

Let n dimensional Riemannian spaces V_n and V_n^* be defined by the fundamental forms

$$ds^2 = g_{ij}\, dx^i\, dx^j \quad \text{and} \quad ds^{*2} = g^*_{ij}\, dx^i\, dx^j$$

respectively with common coordinates x^i. A necessary and sufficient condition that their geodesics correspond with each other is given by

$$\left\{ {i \atop jk} \right\}^* = \left\{ {i \atop jk} \right\} + \delta^i_j\, \psi_k + \delta^i_k\, \psi_j\,, \qquad (\psi_i = \nabla_i\, \psi)\,, \tag{1.1}$$

where ψ is a scalar, $\left\{ {i \atop jk} \right\}$ and $\left\{ {i \atop jk} \right\}^*$ are the Christoffel symbols formed with respect to g_{ij} and g^*_{ij} respectively and ∇_i is the covariant derivative with respect to g_{ij}.

In this paper we shall prove that when V_4 is an S_0, V_4^* is also s. s. This is a generalization of the well known theorem concerning spaces of constant curvature.[2)] Further some other properties concerning the S_0's with corresponding geodesics will be made clear.

§ 2. Fundamental theorem

In this section, first we shall prove the following fundamental theorem:

Theorem [2.1] *The only four dimensional Riemannian spaces with fundamental forms of signature -2 whose geodesics correspond to the geodesics of an S_0 are s. s. space-times.*

Proof. Let the geodesics of a V_4^* correspond to the geodesics of an S_0. We shall take a standard coordinate system for g_{ij} of the S_0. Then the fundamental form of the S_0 is

$$ds^2 = -A\,(r,t)\, dr^2 - B\,(r,t)\,(d\,\theta^2 + \sin^2\theta\, d\,\phi^2) + C\,(r,t)\, dt^2 \tag{2.1}$$