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Orthogonality Relation in the Analysis of Variance II

By

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Let $\mathfrak{C}, \mathfrak{A}_1, \cdots, \mathfrak{A}_p \, (p > 1)$ be non trivial classifications. We shall say that $\mathfrak{C}$ is decomposed into $\mathfrak{A}_1, \cdots, \mathfrak{A}_p$ if the relation

$$P_{\mathfrak{C}} = \sum_{k=1}^{p} P_{\mathfrak{A}_k} \tag{1}$$

holds. Here we may always assume that the $\mathfrak{A}$ are labeled so that $d(\mathfrak{A}_1) \geqq d(\mathfrak{A}_2) \geqq \cdots \geqq d(\mathfrak{A}_p)$. By the result[1] previously established p must be $\geqq 3$. We show (§ 1) that (1) implies

$$d(\mathfrak{A}_1)\, d(\mathfrak{A}_2) \leqq \sum_{k=3}^{p} d(\mathfrak{A}_k), \tag{2}$$

and that if the equality holds in (2), we must obtain

$$P_{\mathfrak{A}_1 \mathfrak{A}_2} = \sum_{k=3}^{p} P_{\mathfrak{A}_k} \tag{3}$$

that is, the interaction $\mathfrak{A}_1 \mathfrak{A}_2$ is decomposed into classifications $\mathfrak{A}_3, \cdots, \mathfrak{A}_p$. Conversely (3) will imply (1) with $\mathfrak{C} = \mathfrak{A}_1 \wedge \mathfrak{A}_2$. Thus a decomposition of an interaction into classifications is regarded as a special case of a decomposition of a classification into classifications. It is our purpose to investigate the structure of such decompositions. The case $p=3$ was considered in our previous paper[2] and we proved that $P_{\mathfrak{C}} = P_{\mathfrak{A}_1} + P_{\mathfrak{A}_2} + P_{\mathfrak{A}_3}$ holds if and only if $\mathfrak{C} = \mathfrak{A}_1 \wedge \mathfrak{A}_2$, and $\mathfrak{A}_1 \mathfrak{A}_2 = \mathfrak{A}_3$. Here the latter relation implies that $\mathfrak{A}_1$, $\mathfrak{A}_2$ are regularly orthogonal and $d(\mathfrak{A}_1) = d(\mathfrak{A}_2) = d(\mathfrak{A}_3) = 1$[3]. After some preliminary researches (§ 1) we investigate the structure of decompositions (1) for the cases $d(\mathfrak{A}_1) = p-2,\ p-3,\ p-4$. It is to be noted that (2) yields $d(\mathfrak{A}_1) \leqq p-2$. Then we apply the results thus obtained to determine the structure of decompositions (1) for $p=4, 5, 6$ (§ 3). § 4 treats the decompositions of interactions. We show there that $P_{\mathfrak{B}\mathfrak{C}\mathfrak{D}} = P_{\mathfrak{A}_1} + P_{\mathfrak{A}_2} + P_{\mathfrak{A}_3} + P_{\mathfrak{A}_4}$ is characterized as the

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1) Cf. [2]. Theorem 22. The numbers in square brackets refer to the list of references at the end of this paper.

2) Cf. [2]. Theorem 23.

3) Cf. [2]. Theorem 9.