JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 17, NO. 1, AUG., 1953

On the Mutual Connectedness of Elements in Lattices

By

Takayuki NÔNO

(Received Jan. 31, 1953)

In this paper we shall first in §1 extend the notion of the connected sets in topological spaces to the case for the lattices with a binary relation, next in §§2 and 3 prove the fundamental theorems concerning this notion, and finally in §4 discuss the relations between this binary relation and a mapping in a complete lattice.

§1. Definitions and axioms

Let L be a lattice. Given in L a binary relation γ, we shall write $x\gamma y$ to express the fact that the elements x and y of L are in the γ-relation in this order. We shall not assume that $x\gamma y$ implies $y\gamma x$. And $x\rho y$ means that $x\gamma y$ and $y\gamma x$ simultaneously. Next we shall define ρ-irreducible elements of L as follows.

Definition. *An element a of L is said to be ρ-irreducible, if a is not expressible as* $a=x\smile y$, $x\rho y$ and $x, y\neq a$.

This notion is an extension of that of connected sets in topological spaces.

As the axioms concerning this γ-relation, we shall consider the following conditions:[1]

(I) If $z\leqq x\smile y$ and $x\rho y$, then $(x\smile y)\frown z=(x\frown z)\smile(y\frown z)$.

(II) If $x\gamma y$, $x_1\leqq x$ and $y_1\leqq y$, then $x_1\gamma y_1$.

(III$_1$) If $x\gamma y_1$ and $x\gamma y_2$, then $x\gamma(y_1\smile y_2)$.

(III$_2$) If $x_1\gamma y$ and $x_2\gamma y$, then $(x_1\smile x_2)\gamma y$.

(IV) If $x\rho x$, then $x=0$. (0 denotes the zero element of L).

(IV*) If $x\leqq y$ and $x\rho y$, then $x=0$.

Remark. It is obvious that (IV) implies (IV*). Under the axiom (II), (IV*) implies (IV). For, by (II) we have $(x\frown y)\rho(x\frown y)$; and since $x\leqq y$, i.e.,

---

1) This concept corresponds to that of "Verkettung" which has been proposed by F. Riesz as a primitive concept of abstract space. F. Riesz, *Stetigkeitsbegriff und abstrakte Mengenlehre,* Atti del IV Congresso Internazionale dei Matematici, 2 (Rome, 1909), pp. 18–24.