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On the Matrix Space

By

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Let $\mathfrak{S}$ be the space of all the complex matrices of degree n with the usual topology, we shall in this paper define the paths in the matrix space $\mathfrak{S}$ and we shall investigate the properties of the paths in the matrix space $\mathfrak{S}$, the general linear group $\mathfrak{M}$ and the special orthogonal group O^+.

§ 1. The paths in the matrix space $\mathfrak{S}$

We shall first consider $\mathfrak{S}$ as a vector space of dimension n^2 and we shall introduce a sort of parallelism into $\mathfrak{S}$ by saying that the vectors MV at the points M of $\mathfrak{S}$ are parallel to each other. (we can define another parallelism by using of VM in place of MV). By the paths in $\mathfrak{S}$ we shall mean the auto-parallel curve $M=M(t)$ (t is a real parameter) with respect to this parallelism, that is, the curve defined by the differential equation:

$$\frac{dM}{dt}=MA, \quad (A \text{ is a constant matrix}).^{1)} \tag{1.1}$$

Then the path through M_0 is given by

$$M(t)=M_0 \exp tA, \quad (M(0)=M_0). \tag{1.2}$$

Let $\mathfrak{A}(M)$ be the set of matrices S such that $SM=0$, (it will be called a left anihilator of M), and let $\rho(M)$ be the rank of the matrix M, then we shall prove the following lemmas.

LEMMA 1. *There exists a matrix X such that $NX=M$ for the given matrices N and M, if and only if $SN=0$ implies $SM=0$, that is, if and only if $\mathfrak{A}(N)\subset\mathfrak{A}(M)$.*

PROOF. If there exists a matrix X such that $NX=M$, then it is clear that $SN=0$ implies $SM=0$. Conversely, we shall assume that $SN=0$ implies

1) The path defined by $\frac{dM}{dt}=MA$ may be called a right path, on the contrary, the path defined by $\frac{dM}{dt}=AM$ may be called a left path. (See Remark 4, p. 60).