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A Note on Lattice Segment

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W. D. Duthie introduced the concept of a segment in a lattice and characterized the modularity and the distributivity of a lattice by it[(1)]. And M. Sholander has, from the axiomatic standpoint, investigated the segments and obtained the three axioms which characterize the segments of a distributive lattice with O and I[(2)].

The purpose of this paper is to generalize the M. Sholander's result and to obtain the axioms which characterize the segments of a lattice with O.

§ 1. Segment of a Lattice L.

In this section, we consider the properties of the segments of a lattice L.

Here, we use the definition of a segment which was used by W. D. Duthie, that is, for any pair a, b of the elements of a lattice L, the set of all elements $x \in L$ which satisfies the condition $ab \leqq x \leqq a+b$ is called the segment joining a and b, and is denoted by the symbol (a, b).

From the above definition of the segment, we have the following lemmas.

(1.1) $$(a, b) \cup (c, d) \subset (abcd, a+b+c+d)$$

PROOF. Suppose $x \in (a, b) \cup (c, d)$. Then element x satisfies the conditions $ab \leqq x \leqq a+b$ or $cd \leqq x \leqq c+d$. So, we have $abcd \leqq x \leqq a+b+c+d$.

Note. Briefly, we write the set $(abcd, a+b+c+d)$ by the symbol $(a,b) \overset{*}{\cup} (c,d)$.

(1.2) Let L be a lattice with O. $(a, b) \subset (p, q)$ if and only if $(O, a) \cap (O, b) \supset (O, p) \cap (O, q)$ and $(O, a) \overset{*}{\cup} (O, b) \subset (O, p) \overset{*}{\cup} (O, q)$.

PROOF. First we prove the necessity. W.D.Duthie has shown that $(a, b) \cap (c, d) =$

1) W. D. Duthie, "Segments of ordered sets," Trans. Am. Math. Soc. vol. 51 (1942) pp. 1–14.

2) M. Sholander, "Tree, lattice, order and betweenness," Proc. Am. Math. Soc. vol. 3 (1952) pp. 369–381.