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A Note on the Relative 2-Dimensional Cohomology Group in Complete Fields with Respect to a Discrete Valuation[1]

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§ 1. Introduction

Let k, K $(k \subset K)$ be $\mathfrak{p}$-adic number fields, and $\mathfrak{o}$, $\mathfrak{O}$ its valuation ring respectively, and $\mathfrak{p}$, $\mathfrak{P}$ its prime ideal respectively. Let D be the relative different of K/k. Then Prof. Y. Kawada[2] has proved the following relations with respect to the relative 2-dimensional cohomology group:

$$(*)\qquad \begin{aligned} &H(\mathfrak{O},\ \mathfrak{o}\,;\ \mathfrak{O}/\mathfrak{P}^r) \cong \mathfrak{O}/\mathfrak{P}^r \quad \text{for} \quad \mathfrak{P}^r \supseteqq D \\ &H(\mathfrak{O},\ \mathfrak{o}\,;\ \mathfrak{O}/\mathfrak{P}^r) \cong \mathfrak{O}/D \quad \text{for} \quad \mathfrak{P}^r \subseteqq D. \end{aligned}$$

Now let k be a complete field (with respect to a discrete valuation) of such a quotient field as, in its integral domains, the fundamental theorem of the multiplicative ideal theory holds. Let K be a separable extension of finite degree over k. Let $\mathfrak{o}$, $\mathfrak{O}$ be its valuation ring respectively, and $\mathfrak{p}$, $\mathfrak{P}$ its prime ideal respectively. Let D be the relative different of K/k.

Our aim is to prove that $(*)$ relation is satisfied in the case of $\mathfrak{O}=\mathfrak{o}[\theta]$, and that, using this fact, we can give a definition of the relative different of K/k different from the usual one.

In the case of $\mathfrak{p}$-adic number fields,[3] let K' be the inertia field of K/k, and $\mathfrak{O}\cap K' = \mathfrak{O}'$. Then $\mathfrak{O}'=\mathfrak{o}[\zeta]$, and, moreover, since the number of the elements of the residue class field of K by $\mathfrak{P}$ is finite, we can take ζ such that $\zeta^{|N(\mathfrak{P})|-1}=1$ where $|N(\mathfrak{P})|$ is the absolute value of the absolute norm of $\mathfrak{P}$. In addition, we can set $\mathfrak{O}=\mathfrak{O}'[\Pi]$ where Π is a prime element of $\mathfrak{P}$ in $\mathfrak{O}$. These two facts are essentially utilized for the proof of $(*)$ relation, but, in our case, they are not always true.

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2) Y. Kawada: On the derivations in number fields, Annals of Math. **54** (1951), pp. 310–314.

3) Y. Kawada, l. c.