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Convergence of Numerical Iteration in Solution of Equations

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§ 1. Introduction

Let R be a *linear normed space* and F be a *complete subset of* R. Let T be a functional defined on F such that $T(F) \subset R$. We assume that T *is Lipschitz bounded, namely that there exists a positive constant* K *such that*

$$\| Tf_1 - Tf_2 \| \leqq K \| f_1 - f_2 \|$$

for any $f_1, f_2 \in F$.

In F, let us consider the equation

$$x = Tx. \tag{1.1}$$

We assume that

(i) $K < 1$;

(ii) *for a selected* $x_0 \in F$, $x_1 = Tx_0$ *belongs to* F;

(iii) *the sphere* $S\{h : \|h - x_1\| \leqq \frac{K}{1-K} \|x_1 - x_0\|\}$ *is contained in* F.

L. Collatz [1] has shown that the iteration $x_{n+1} = Tx_n$ $(n = 0, 1, 2, \cdots)$ can be continued idefinitely and the sequence $\{x_n\}$ converges to a certain limit $\bar{x}$ which gives a unique solution of (1.1) in F. But, when x is a numerical quantity, there arises an error in computation of Tx, consequently, in numerical iteration, the obtained sequence is not the sequence $\{x_n\}$ determined by $x_{n+1} = Tx_n$, but the numerical sequence $\{x_n^*\}$ determined by $x_{n+1}^* = T^* x_n^*$, where T^* is a certain approximate functional of T. Then, as is shown in this paper, *the numerical sequence* $\{x_n^*\}$ *does not necessarily converge contrary to convergence of the true sequence* $\{x_n\}$. Then, in order to seek for the solution of (1.1), *at what step the iteration process should be stopped? When the iteration process is stopped at the favorable step in this sense, with how*

1) L. Collatz, *Einige Anwendungen funktionalanalytischer Methoden in der praktischen Analysis*, Z. Angew. Math. Phys., **4**, 327-357 (1953).