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Convexification of Locally Connected Generalized Continua

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1. Introduction.

By a generalized continuum we mean a locally compact, connected, separable, metric space. If a generalized continuum is locally connected and compact, it is called a continuous curve or sometimes a Peano continuum. We say that a space R having a metric $d(x, y)$ is finitely compact if, for each positive number α and a point p in R, the closure of the set $\{x/d(p, x) < \alpha\}$ is compact. Finitely compact spaces are complete. It is known that each pair of points in a continuous curve can be joined by an arc. A metric $d(x, y)$ is called convex if p and q are points in R, there is a point r in R such that $d(p, r)=d(q, r)=d(p, q)/2$. If $d(x, y)$ is a convex metric for a compact (or more generally, complete) space R, then for each pair of points p, q of R there is an arc pq in R from p to q such that pq is isometric to a straight line interval. (Such arcs are called segments.)

In 1928, K. Menger [4] had proposed the following question: Whether or not each continuous curve has a convex metric that preserves its topology? By R. H. Bing [1] and E. E. Moise [5] the question was answered in the affirmative.

It this paper, we shall consider the convexification problem for the case where the space is locally compact, not necessarily compact, and prove the following theorem:

THEOREM. *Each locally connected generalized continuum has a convex metric and each pair of points can be joined by a segment.*

The method of this theorem is a modification of the method used by Bing.

2. Definitions.

For convenience we shall arrange the definitions used in this paper. Most of them were introduced in the papers [1] and [2].

Property S. A set has *property* S if, for each positive number ε, the set is