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Topologies on Rings of Operators

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In the course of the investigations of the properties of a ring $\mathbb{M}$ of operators on a Hilbert space six different topologies have been introduced into the ring $\mathbb{M}$ by various writers ([**12**], [**13**], [**4**], [**5**], [**9**]) : *uniform*, $\sigma(\mathbb{M}, \mathbb{M}^*)$, *ultrastrong*, *ultraweak*, *strong* and *weak topologies*. In comparing the different topologies we use the words "stronger than" to mean "at least as strong" and give this meaning to the symbol $>$. Among these topologies the following relations hold :

$$\begin{array}{ccccc} \text{Uniform top.} & > & \text{Ultrastrong top.} & > & \text{Strong top.} \\ \vee & & \vee & & \vee \\ \sigma(\mathbb{M}, \mathbb{M}^*) & > & \text{Ultraweak top.} & > & \text{Weak top.} \end{array}$$

It is the main purpose of this paper to study the conditions for any assigned two topologies on a ring $\mathbb{M}$ among these six ones cited above to coincide.

§ **1** includes the preliminaries to the rest of the paper. Although most of the results of §1 can be found in the literature as the references indicate, the proofs are given them for the sake of completeness of our treatment. In §**2** we give the conditions of equivalence of two topologies "ultraweak" and "weak" (or "ultrastrong" and "strong") on $\mathbb{M}$. The results obtained are closely related to those of Dye [**7**], Griffin [**9**] and Dixmier [**5**]. § **3** is devoted to the discussions of some properties of cyclic projections. In § **4** we show that any two topologies on $\mathbb{M}$ (besides the cases mentioned above) coincide if and only if $\mathbb{M}$ is finite-dimensional.

The results of the papers of Dixmier [**1**], [**2**] and [**3**] are assumed to be known and will be used without further reference.

§ I. General remarks on the spatial properties of a rings of operators

In what follows, unless otherwise stated, $\mathbb{M}$ stands for a ring of opreators (containing the identity operator I) on a Hilbet space H. Let K be a Hilbert