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Dimension Functions on Certain General Lattices

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Introduction

There have been developed in the literature two kinds of the theory of dimensionality in lattices, that is, one is the theory in the continuous geometries (von Neumann [13], Halperin [5], Iwamura [7]) and the other is the theory in the operator rings (Murray and von Neumann [12], Segal [15]) and AW*-algebras (Kaplansky [8], Sasaki [14]). The main purpose of the present paper is to investigate a dimensionality in certain general (not necessarily modular) lattices so that all the above cases may be treated from the uniform standpoint.

Before describing the outline of this paper, it is convenient to explain the above theories in some more details.

(I) In the theory of continuous geometries, von Neumann [13, I] introduced dimensionality by perspectivity. He constructed numerical dimension functions in the irreducible case, and in the reducible case Iwamura [6] introduced dimension functions whose ranges are sets of continuous functions on the Boolean space which represents the center Z of the lattice.

In continuous geometries, generalizing the idea of Halperin [5], Iwamura [7] introduced the concept of the p-relation which means a relation $a \sim b$, satisfying the following conditions:

(1) $a \sim b$ is an equivalence relation;

(2) if a and b are perspective, then $a \sim b$;

(3) (complete additivity) if $a_\alpha \sim b_\alpha$ for every $\alpha \in I$, then $\oplus a_\alpha \sim \oplus b_\alpha$, where $\oplus$ means the l. u. b. of an independent system in von Neumann's sense [13, I, p. 9];

(4) if $a \sim \oplus b_\alpha$, then there exists a decomposition $a = \oplus a_\alpha$ with $a_\alpha \sim b_\alpha$;

(5) (finiteness) there exists no pair of elements a, b such that $a \sim b < a$.

He defined the relative center with respect to the given p-relation as the set of all z such that $a \sim z$ implies $a = z$. A dimensionality is induced by a p-relation, and the relative center plays the rôle of the center Z.