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On the Behavior of Paths of the Analytic Two-dimensional Autonomous System in a Neighborhood of an Isolated Critical Point (Continued)

Hisayoshi Shintani

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§ 1. Introduction

Given an analytic two-dimensional autonomous system

$$(1) \qquad \frac{dx}{dt}=X(x, y), \qquad \frac{dy}{dt}=Y(x, y),$$

for which the origin is an isolated critical point.

In the previous paper [1][1], to study the behavior of paths of (1) in a neighborhood of the origin, we considered the differential equation

$$(2) \qquad \frac{dy}{dx}=\frac{Y(x, y)}{X(x, y)}$$

and studied the classification of the integral curves of (2) tending to the origin in the fixed directions, namely we sought for all possible types of such integral curves tending to the origin that they may be represented by the equations of the form $y=y(x)$ or $x=x(y)$.

In this paper, making use of the results obtained there, for the case where there exists at least one integral curve tending to the origin in a fixed direction[2], we study the full configuration of the paths of (1) around the origin.

When $X(x, y)\equiv 0$ in a neighborhood of the origin, all the integral curves in that neighborhood are represented by the equation $x=const.$, so the configuration of the paths near the origin is completely known. Hence, in the sequel, we consider only the case where $X(x, y)\not\equiv 0$ in any small neighborhood of the origin.

At first, a sufficiently small circular neighborhood of the origin is divided into a finite number of sectors by the curve $X(x, y)=0$, and next, each sector is further divided into some subsectors by some kinds of integral curves. Then, after some considerations about the integral curves in each subsector, the full configuration of the paths around the origin can be known.

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1) The numbers in square brackets refer to the references listed at the end of this paper.

2) As is readily seen, whether or not there exists at least one integral curve tending to the origin in a fixed direction can be decided by a finite number of steps mentioned in the previous paper.