J. SCI. HIROSHIMA UNIV., SER. A
VOL. 23, NO. 2, DECEMBER, 1959

On the Behavior of Paths of the Analytic Two-dimensional Autonomous System in a Neighborhood of an Isolated Critical Point

Hisayoshi SHINTANI

(Received April 30, 1959)

§ 1. Introduction

The object of this paper is to study the behavior of paths of an analytic two-dimensional autonomous system in a neighborhood of an isolated critical point.

First I. Bendixson [1][1] treated this problem systematically. He showed that, by a finite number of quadratic transformations, one might reduce the study of such a problem to that of the case where the critical point is simple or of the so-called Bendixson type.

Later in 1928, M. Frommer [2] introduced the notions of the orders and magnitudes of curvature of the paths tending to a critical point and, making use of these notions and of the exceptional directions, he gave a systematic method to determine the behavior of paths in a neighborhood of a critical point. But, as is noticed by V. V. Nemytzkii and V. V. Stepanov [3], his method seems to be not sufficient.

Recently S.Lefschetz [4] gave a step-by-step process to reduce the study of the behavior of paths in a neighborhood of a critical point to the study of the paths in the case where the critical point is simple or of the Bendixson type. At this juncture, he made use of the coordinate transformations so often that the behavior of the paths of the initial system is not always clear.

In this paper, making use of a Newton polygon, exceptional directions and Keil's theorem, we give a simple step-by-step process to obtain a local phase portrait of the paths near the critical point without using any coordinate transformation except for a simple one used only once.

§ 2. Preliminaries.

2.1 Exceptional directions and Keil's theorem.

Given a continuous autonomous system

$$\frac{dx}{dt}=P(x, y), \quad \frac{dy}{dt}=Q(x, y), \tag{1}$$

---

1) The numbers in square brackets refer to the references listed at the end of this paper.