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A Remark to Existence and Uniqueness of Certain Stable Solutions of a Weakly Nonlinear System

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1. Introduction.

In this note, we consider the n-dimensional system

$$(1) \qquad x' = Ax + f(t, x) \qquad \left(' = \frac{d}{dt}\right)$$

such that

$1°$ the $k (k \leqq n)$ characteristic roots $\lambda_i (i=1, 2, \cdots, k)$ of the constant matrix A have negative real parts;

$2°$ the remaining $(n-k)$ characteristic roots λ_i $(i=k+1, \cdots, n)$ have non-negative real parts;

$3°$ for a certain $\Delta > 0$,

$$(2) \qquad f(t, x) = (|x|^{1+\Delta})^{1)}$$

uniformly in t as $|x| \to 0$.

For such a system, in the book of E. A. Coddington and N. Levinson [1], there is stated a theorem[2] as follows:

When λ_i's $(i=1, 2, \cdots, k)$ are arranged so that $R\lambda_i \leqq R\lambda_{i+1}$ $(i=1, 2, \cdots, k-1)$, if $x=\varphi(t)$ is a solution of (1) *and it holds that*

$$\limsup_{t\to\infty} \frac{\log|\varphi(t)|}{t} = b < 0,$$

then there exist integers p and q $(1 \leqq p \leqq q \leqq k)$ *such that*

$$R\lambda_{p-1} < R\lambda_p = R\lambda_{p+1} = \cdots = R\lambda_q = b < R\lambda_{q+1}$$

and there exists a $\delta > 0$ *and a solution*

$$(3) \qquad x = \psi(t) = \sum_{j=p}^{q} Q_j(t) e^{\lambda_j t}$$

of $x' = Ax$ *such that*

$$(4) \qquad \varphi(t) = \psi(t) + O(e^{(b-\delta)t})$$

as $t \to \infty$. *Here $Q_j(t)$'s are column vectors each component of which is a polynomial in t.*

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1) $|x|$ means $\sum_{i=1}^{n} |x^i|$, where x^i's are the components of the vector x. In the sequel, we use this convention.

2) In the sequel, we call this theorem the theorem of Coddington and Levinson.