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*On the Lattice of Projections of a Baer *-ring*

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Introduction

In my previous paper [8] it has been shown that in any complete lattice with independence relation " $\perp$ " and equivalence relation " $\sim$ " defined by the axioms $(1,\alpha)$—$(1,\zeta)$ and $(2,\alpha)$—$(2,\zeta)$ respectively, there can be introduced the dimension functions which satisfy the conditions of Definition 5.2 of [8]. As shown there ([8], §2), both continuous geometries and the lattice formed by the projections of any AW*-algebra are of this category.

In any Baer *-ring A, the lattice L of its projections satisfies the axioms $(1,\alpha)$—$(1,\zeta)$ if $e\perp f$ stands for $ef=0$ $(e, f\in L)$, and we can define in L two kinds of equivalence: algebraic- and *-equivalences. The main purpose of this paper is to search for the conditions under which these equivalence relations satisfy the axioms $(2,\alpha)$—$(2,\zeta)$, in order to introduce the dimension functions on L with requisite properties above referred to.

For this purpose, we shall replace the axiom of complete additivity $(2,\delta)$ by the weaker axioms: the axiom of finite additivity $(2,\delta_1)$ and that of complete additivity in the restricted sense $(2,\delta_2)$ whose precise statements are given in §1. We show (Theorem 1.1) that $(2,\delta)$ follows from $(2,\delta_1)$ and $(2,\delta_2)$ by the aid of the other axioms if we further assume the axioms $(2,\bar{\gamma})$ and $(2,\eta)$ below (§1), which are clearly verified in both continuous geometries and AW*-algebras (see [8], p. 218).

In §2, it is proved (Theorem 2.1) that algebraic equivalence (resp. *-equivalence) satisfies the axioms $(2,\alpha)$—$(2,\zeta)$ if A satisfies the condition (a) (resp. (a^*)), which means that any two perspective projections are algebraically equivalent (resp. *-equivalent). And it is proved (Theorem 2.2) that if moreover A is finite then L is a continuous geometry, as in the case of an AW*-algebra (Kaplansky [3], Theorem 6.5).

The Baer *-ring which appears in Kaplansky [4] (resp. [5]) satisfies the condition $(\bar{a})$ (resp. $(\bar{a}^*)$) (Remark 2.2) which is stronger than (a) (resp. (a^*)). In §3, the reduction theory of a finite Baer *-ring A satisfying $(\bar{a})$ (resp. $(\bar{a}^*)$) is discussed by the aid of the reduction theory of continuous geometry. The main result (Theorem 3.1) is that the factor ring A/I of A by a maximal restricted ideal I is also a finite Baer *-ring satisfying $(\bar{a})$ (resp. $(\bar{a}^*)$) and is irreducible, and the projections of A/I from a simple